Solutions

1. Ideally $V_i = i\,V_{ref}\,2^{-N}$, $i = 0, 1, \dots 2^N - 1$

Actually $V_{ia} = V_{ref} \sum_{k=1}^{i} R_{ave}(1+\Delta_k) \Big/ (2^N R_{ave})$

where $R_{ave} = \sum_{k=1}^{2^N} R_k / 2^N$

and $\Delta_k = \Delta R_k / R$

$$\text{Max INL} = (V_i - V_{ia})_{max} = V_{ref}\,2^{-N}\left[i - \sum_{k=1}^{i}(1+\Delta_k)\right]$$

In the middle of the string, $i = 2^{N-1}$

$$INL_{max} = V_{ref}\,2^{-N}\,i\,\Delta_k = (V_{ref}/2)\,\Delta_k$$

$$LSB = V_{ref}\,2^{-N}$$

$$|\Delta_k| < 2^{-N} = 2^{-10} \sim 10^{-3}$$

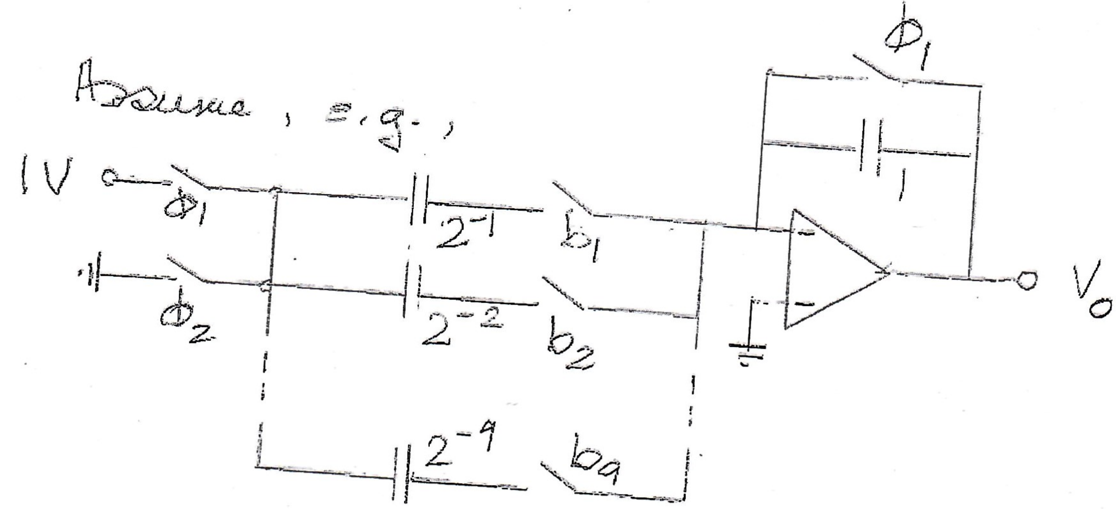
For $\Delta_k = 0.15 = 2^{-(ENOB)}$

$2^{ENOB} = 6.6\dot{6}$; $ENOB = \lg 6.6 / \lg 2 \cong 2.73$

Solutions.

2.

Assume, e.g.,



$V_o(n) = \sum_{i=1}^{9} b_i(n)\, 2^{-i}$ for ideal caps

If the input caps have relative ~~error~~ r_i

$V_o(n) = \sum_{i=1}^{n} b_i(n)\,(1+r_i)\,2^{-i}$

Major carry DNL max. for $10000\ldots \leftrightarrow 0111\ldots$, if $C_{max} \Rightarrow (1+r_{max})\,2^{-1}$, and all others become $(1-r_{max})\,2^{-i}$. Then

$DNL_{max} = r_{max} \sum_{i=1}^{9} 2^{-i} = r_{max}(1-2^{-9})$ (V)

$1\ LSB = 2^{-9}$ V, so

$DNL_{max} \cong 2^{9} r_{max}$ (LSB) $\overset{!}{=}$ 0.5 (LSB)

So $r_{max} \cong 2^{-10} \cong 0.9766 \times 10^{-3} \sim 0.1\%$

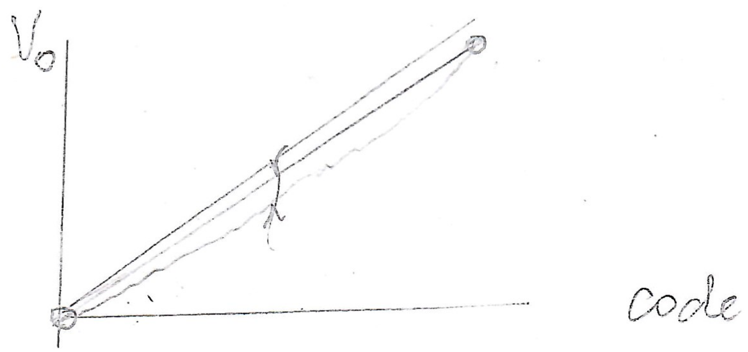
For 10-bit case, rmax ~ 2^(-11) = 0.05%.

For <u>absolute</u> INL with r_{max}, 111...1

$$INL^{a}_{max} = DNL_{max} \cong 0.5(LSB) \cong 0.977(mV)$$

<u>Endpoint</u> INL max occurs for 1000...0

$$INL^{e}_{max} = 2^{-1} r_{max} \cong 0.488\ mV \cong 0.25\ LSB$$



INL(max) = 1/2*DNL(max)

Since rmax was calculated for DNL(max) = 0.5*LSB, INL(max) = 0.25*LSB regardless of N.