ECE 627

MIDTERM EXAMINATION

May 11, 2012

Open book

In the segmented 2*N*-bit DAC circuit shown, the value of the bridge capacitor was chosen as C_0 (the unit capacitor of the array), instead of the fractional capacitance derived in class. This allows easier layout, but also changes the signals in the array, and thus introduces an error into the operation of the DAC. This problem deals with this error.

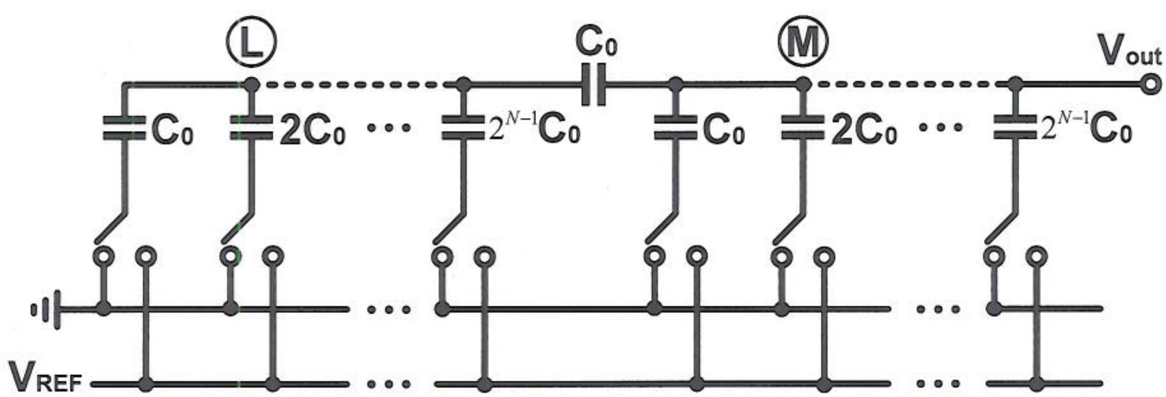
Initially, the top and bottom plates of all capacitors are at ground potential.

1. Find the capacitance C_L between node **L** and ground, and the capacitance C_M between node **M** and ground.

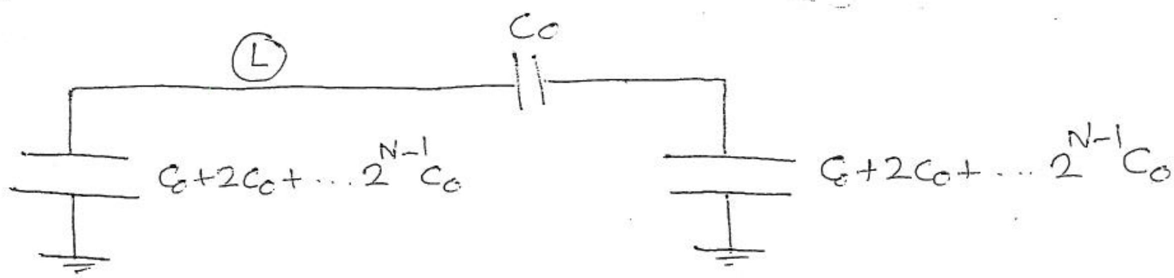
2. Find the node voltages V_L and V_M when the bottom plate of the capacitor $2C_0$ in the left-side (LSB) array is switched from ground to V_{REF}.

3. Repeat the calculation with the bottom node of the MSB array capacitor $2C_0$ switched to V_{REF}.

4. How significant is the voltage error in V_L and V_M due to the change in the value of the bridge capacitor to C_0?

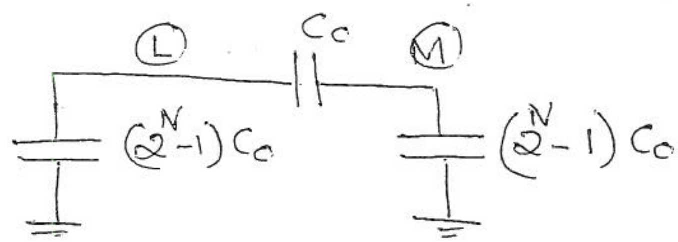


1) Node (L) can be equivalently shown as



Now $C_o + 2C_o + \dots 2^{N-1} C_o = (2^N - 1) C_o$

$\Rightarrow$



$\Rightarrow (2^N - 1) C_o + C_o \,//\, (2^N - 1) C_o$ → Parallel operation $\frac{C_A C_B}{C_A + C_B}$ But series connection

$$\Rightarrow C_L = (2^N - 1) C_o + \frac{C_o * (2^N - 1) C_o}{\cancel{C_o} + 2^N C_o - \cancel{C_o}}$$

$$C_L = (2^N - 1) C_o + \frac{(2^N - 1) C_o}{2^N}$$

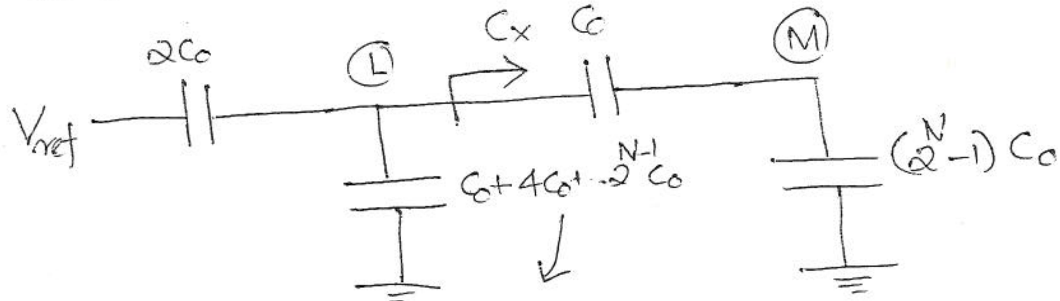
$$\Rightarrow C_L = 2^N C_o - \cancel{C_o} + \cancel{C_o} - 2^{-N} C_o$$

$$\Rightarrow C_L = \left(2^N - 2^{-N}\right) C_o$$

By symmetry

$$\boxed{C_M = C_L = \left(2^N - 2^{-N}\right) C_o}$$

2) Equivalent circuit when bottom plate of capacitor $2C_o$ is switched to V_{ref}



$$(2^N-1)C_o - 2C_o$$
$$= (2^N-3)\,C_o$$

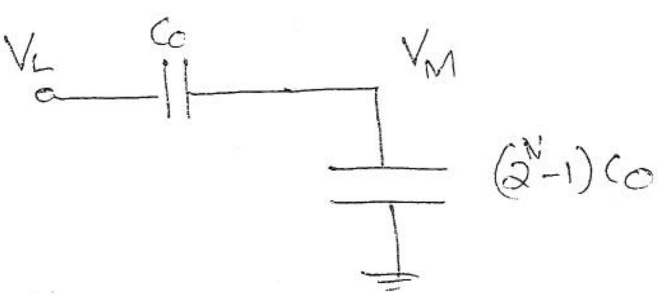
$$V_L = \frac{2C_o}{2C_o + C_x + (2^N-3)\,C_o} \cdot V_{ref} = V_{ref} \times \frac{2C_o}{(2^N-1)\,C_o + C_x}$$

$$C_x = \frac{C_o \times (2^N-1)\,C_o}{C_o + (2^N-1)\,C_o} = \left(1-2^{-N}\right) C_o$$

$$\Rightarrow V_L = V_{ref} * \frac{2C_o}{(2^N-1)\,C_o + \left(1-2^{-N}\right) C_o}$$

$$= V_{ref} * \frac{2\cancel{C_o}}{\left(2^N-2^{-N}\right)\cancel{C_o}}$$

$$\Rightarrow V_L = V_{ref} * \frac{2}{2^N-2^{-N}}$$

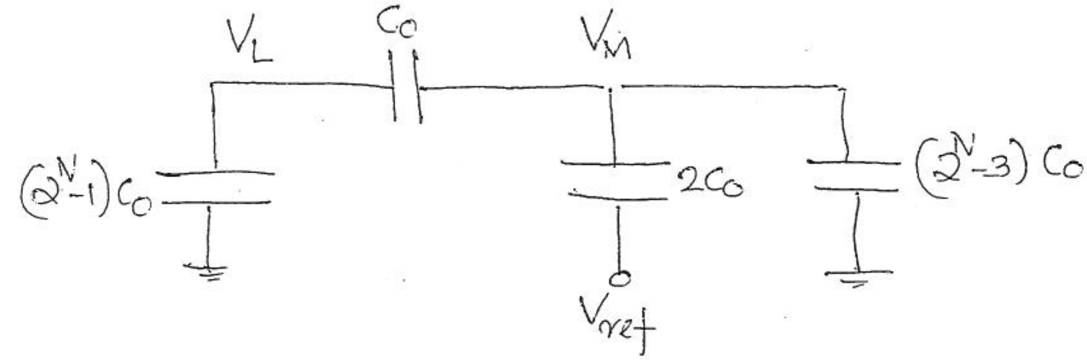


$$V_M = V_L * \frac{C_o}{C_o + (2^N-1)\,C_o}$$

$$\Rightarrow V_M = \frac{V_L}{2^N}.$$

So $$\boxed{\begin{aligned} V_L &= V_{ref} * \frac{2}{2^N - 2^{-N}} \\ V_M &= V_{ref} * \frac{2}{2^{2N} - 1} \end{aligned}}$$

3) When $2C_0$ of MSB array is switched to V_{REF}



This is exactly symmetrical to the previous case except that V_L and V_M are interchanged

$\therefore$ $$\boxed{\begin{aligned} V_M &= V_{ref} * \frac{2}{2^N - 2^{-N}} \\ V_L &= V_{ref} * \frac{2}{2^{2N} - 1} \end{aligned}}$$

4) If the bridge capacitor were exact value [the fractional one], we would ~~exeq~~ expect V_L and V_M in case ② to be

$$V_L' = V_{ref} \cdot \frac{2}{2^N} \qquad LSB = \frac{V_{ref}}{2^{2N}}$$

$$= \frac{V_{ref}}{2^{N-1}}$$

and $V_{out}' = V_M = V_{ref} \cdot \dfrac{2}{2^{2N}}$ [ie 2 LSB]

But since we had put bridge capacitor as

C_o

$$V_{out} = V_M = V_{ref} \times \frac{2}{2^{2N}-1}$$

$$= V_{ref} \cdot \frac{2}{2^{2N}\left[1 - 2^{-2N}\right]}$$

$$V_{out} \approx V_{out}' \left[1 + 2^{-2N}\right]$$

assuming $2^{2N} \gg 1$

So error $\boxed{\dfrac{\Delta V_{out}'}{V_{out}'} = 2^{-2N}}$ which is negligible for large value of N.

For 12 bit DAC, this error $= 0.024\,\%$.

For $V_L = V_{ref} \cdot \dfrac{2}{2^N - 2^{-N}}$

$$V_L' = V_{ref} \cdot \frac{2}{2^N}$$

$$V_L = V_{ref} \cdot \frac{2^{N+1}}{2^{2N} - 1}$$

$$\approx \frac{V_{ref} \cdot 2^{N+1}}{2^{2N}} \cdot \left(1 + 2^{-2N}\right)$$

$$\approx V_L' \left(1 + 2^{-2N}\right)$$

$\Rightarrow$ Error remains the same and of small value.