ECE 627

Midterm Examination

May 10, 2013

Open book

In the SAR ADC shown on the next page, initially S_1 and S_{21} are closed, as well as S_3, S_5, S_7, ... , S_{19} . Then S_1 and S_{21} open, and S_2 closes to sample V_{in} at node A. Afterwards, the switches $S_3 - S_{20}$ are used to generate the comparison voltages V_A at node A to find the bits b_i of the output word. The element values are C = 0.4 pF, C_s = 0.1 pF and R = 1 kΩ. V_{ref} = 3 V.

a. What is the resolution of the ADC?

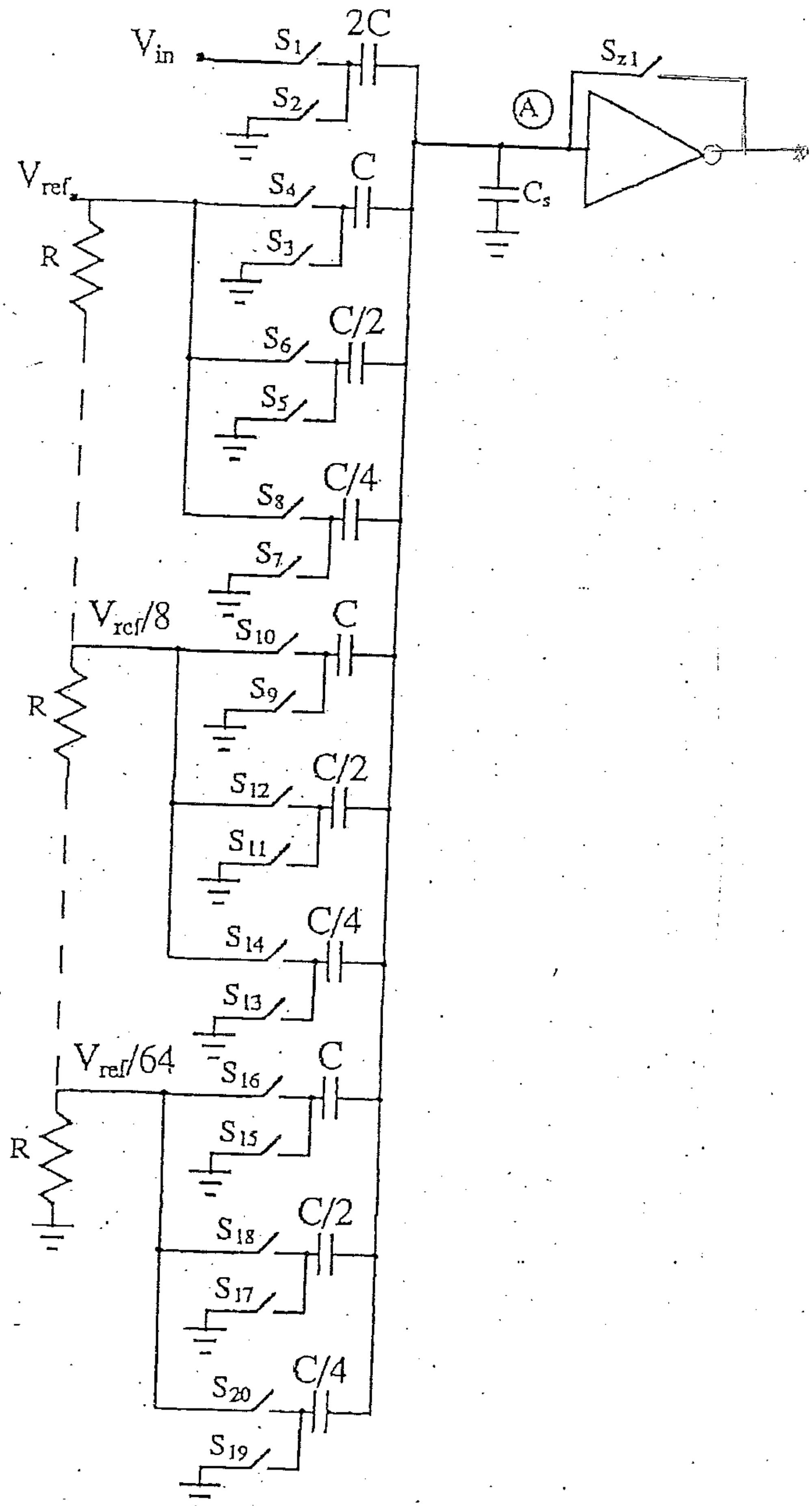
b. What are the MSB and LSB voltage steps in V_A?

c. Find the expression for V_A in terms of V_{in}, b_i, C and V_{ref}.

d. What will be the value of the voltage V_A after the first 3 MSB cycles if $V_{in} = 0.8V_{ref}$?

e. What is the worst-case error in V_A if the resistors have a 1% matching error?

f. What is the worst-case DNL error if the capacitors have a 0.2% matching error?

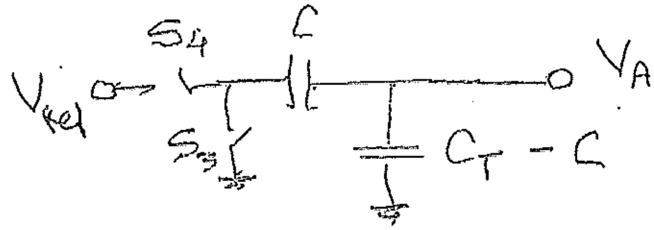


V_{in}
S_1
2C
S_2
S_{z1}
A
C_s
V_{ref}
S_4
C
S_3
R
S_6
C/2
S_5
S_8
C/4
S_7
$V_{ref}/8$
S_{10}
C
S_9
R
S_{12}
C/2
S_{11}
S_{14}
C/4
S_{13}
$V_{ref}/64$
S_{16}
C
S_{15}
R
S_{18}
C/2
S_{17}
S_{20}
C/4
S_{19}

Solutions

(a) By inspection $N = 9$

(b) MSB step $(S_3 - S_4)$:



$$C_T = 2C + C_S + 3C(1 + 0.5 + 0.25)$$

$$= 7.25C + C_S = 3\,pF$$

$$\Delta V_{A_{MSB}} = \frac{C}{C_T} V_{ref} = 0.4\,V$$

$$\Delta V_{A_{LSB}} = \frac{C/4}{C_T} \frac{V_{ref}}{64} = 1.5625\ mV$$

$$\Delta V_{A_{MSB}} / \Delta V_{A_{LSB}} = 256 = 2^8$$

(c) $\hat{V}_A = (V_{ref} C / C_T)(b_1 + b_2 2^{-1} + \dots + b_9 2^{-8})$

For $V_{in} = 0$ ↗

$$= 0.4 \sum_{i=1}^{9} b_i 2^{-i+1} = 0.8 \sum_{i=1}^{9} b_i 2^{-i}\ V$$

$$\therefore V_A = \hat{V}_A - \frac{2C}{C_T} V_{in} \cong 0.8 \sum_{i=1}^{9} b_i 2^{-i} - 0.266\, V_{in}$$

(d) $V_A = 0.8(0.5 + 0.25) - 0.64 = -40\ mV$

(e) Worst case: R's below $V_{ref}/8$ too small, above too large (or vice versa). Then, the string current is $i = V_{ref}/(56.56 + 7.92)$

$i \doteq 46.526$ mA

$V_8 \triangleq V_{ref}/8 = 0.375V \rightarrow 0.3685$ V

$V_{64} = V_{ref}/64 \cong 0.0469V \rightarrow 0.04606$ V

In the worst case, all caps at $V_{ref}/8$ and $V_{ref}/64$ are active, so the error in V_A is

$$\Delta V_A = \frac{C}{C_T}(1 + 0.5 + 0.25)(\Delta V_8 + \Delta V_{64}) =$$

$$\Delta V_A = \frac{0.4}{3} 1.75 (6.5 + 0.24) \text{ mV} \cong 1.713 \text{ mV} > 1\text{LSB}$$

(f) Worst case: $100 \cdots 0 \leftrightarrow 011 \cdots 1$.

Assume MSB cap $C_{MSB} \rightarrow 1.002C$, and all others $0.998 C_{nom}$. Then $C_T = 0.998(6.25C + C_s) + 1.002C = 2.9956$ pF. Change in V_A value:

$$\Delta V_A = V_{ref}\left[\frac{1.002C}{C_T} - 0.998(2^{-1} + \dots 2^{-8})\frac{C}{C_T}\right]$$

$$\Delta V_x = V_{ref} \frac{C}{C_T} \left[1.002 - 0.998 \times (1 - 2^{-8}) \right]$$

$$\cong 0.4006 \times 0.0079 \cong 3.164\ mV \sim 2\ LSBs$$