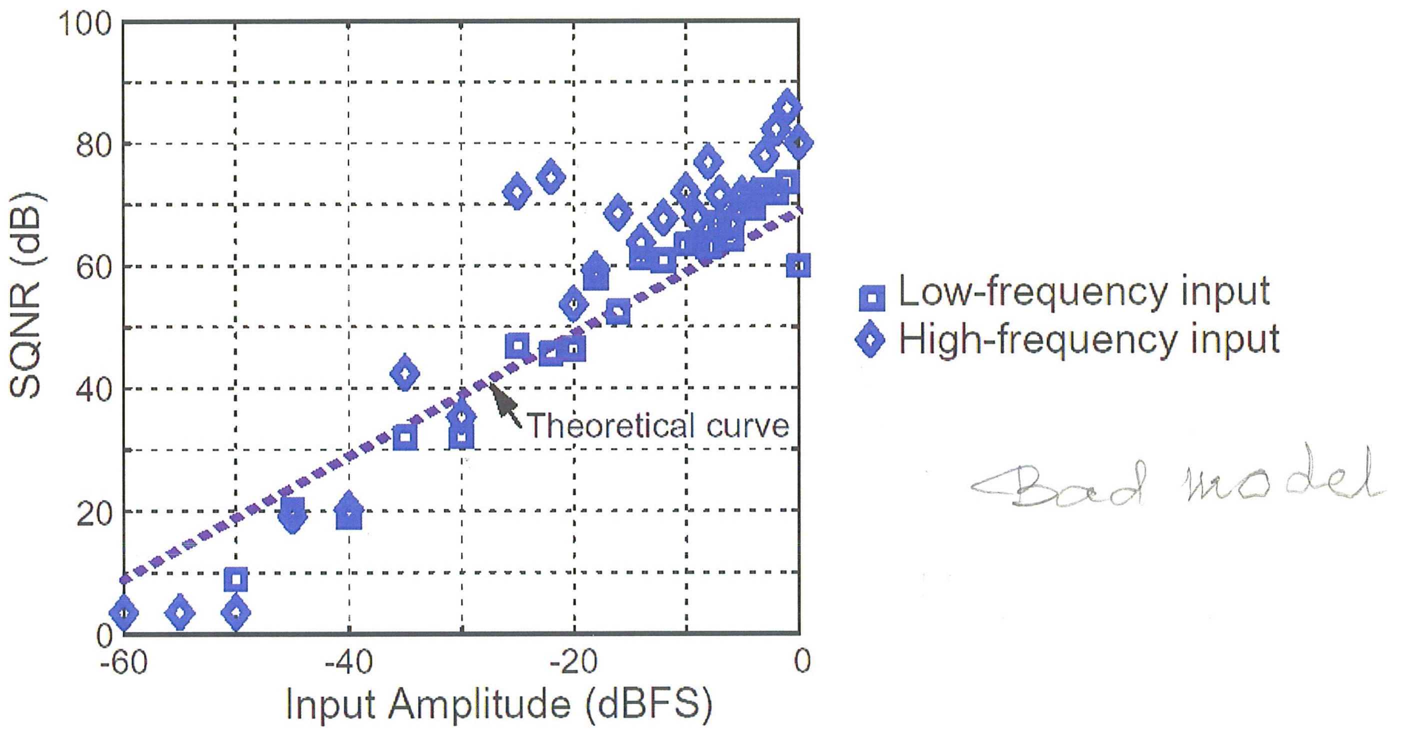
Simulation of MOD1 (2)

- SQNRs for different frequencies:



Sine wave inputs

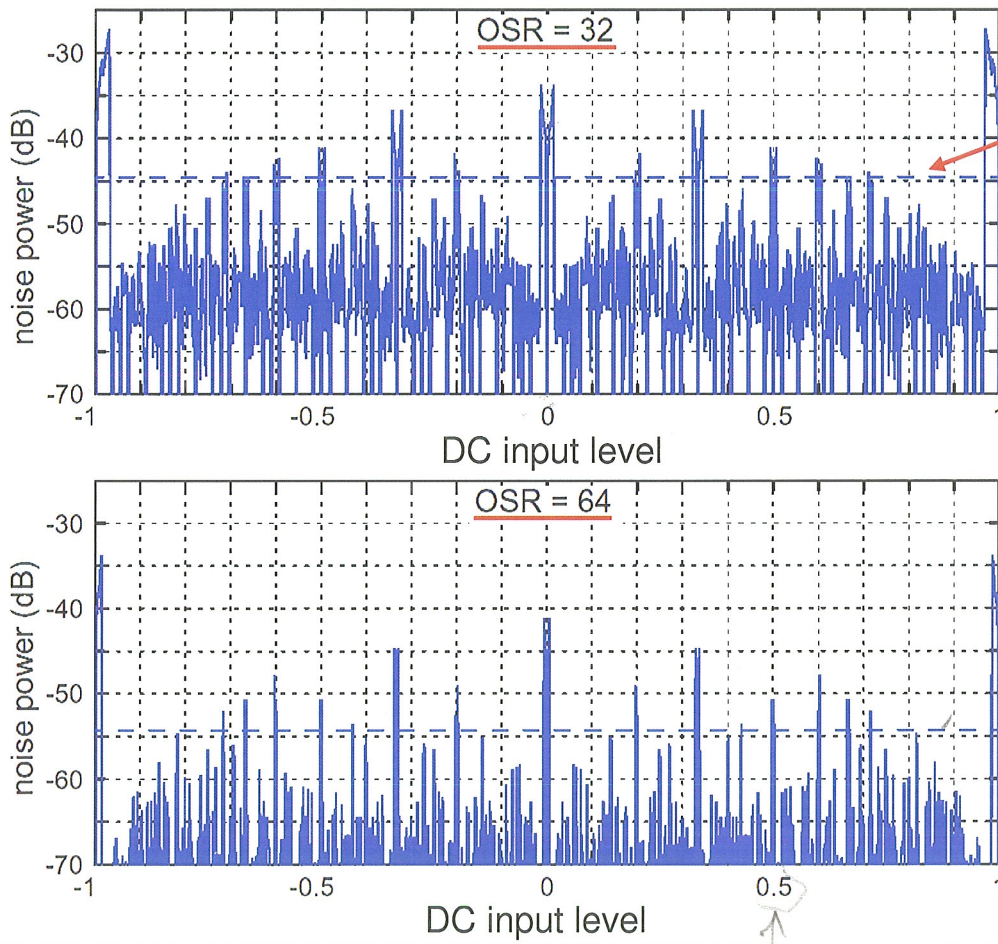
Simulation of MOD1 (3)

- In-band quantization noise power:

0.99...98

111....1-111...-11

$V_{ref} = \pm 1$



mean square of inband noise

idle tones (limit cycles)

u = 1/2

MOD1 Under DC Excitation (1)

- Idle tones:

$$s(n) = \sum_{i=1}^{n} v(i)$$

Σ $\quad y(n) = y(n-1) + u - v(n-1)$ $\quad$ (1/2, 1/2, −1)

Q $\quad v(n) = \mathrm{sgn}(y(n))$, $\mathrm{sgn}\,0 = 1$

$$y(n) = y(n-1) + u - \mathrm{sgn}(y(n-1))$$

0.5001

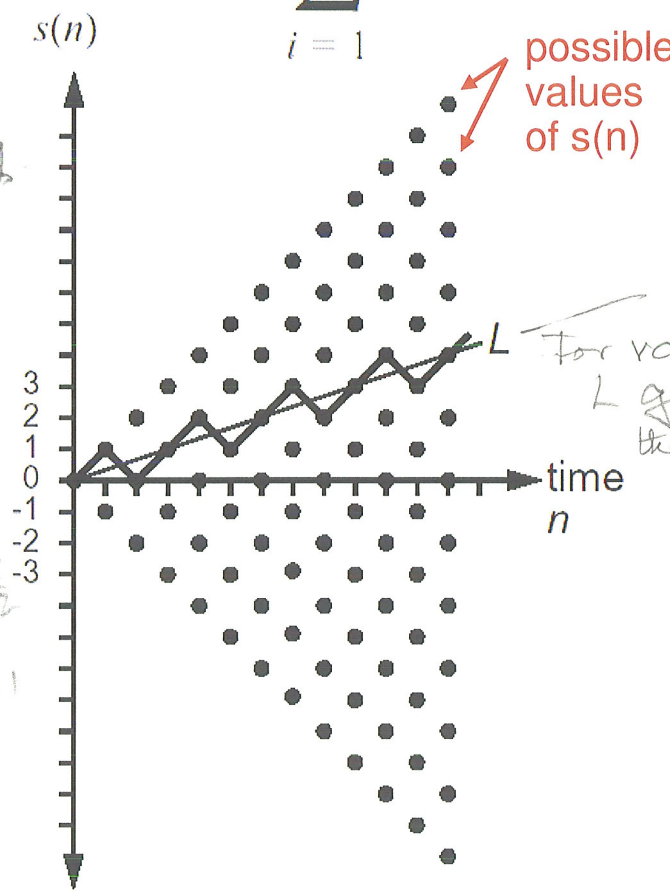
- u = y(0) = ½: $\quad V_{ref} = 1$

Tone at $f_s/4$

n	0	1	2	3	4
$y(n)$	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	$\frac{1}{2}$
$v(n)$	1	1	-1	1	1

$0 \quad -\frac{1}{2}$

$1 \quad -1$



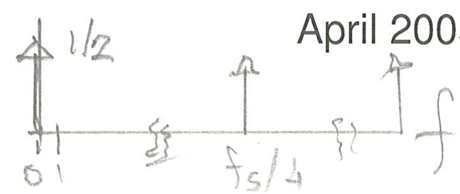
slope = u

For rational u, L goes thru the lattice points

- For u = 0.01, tones at k·f_s/200!

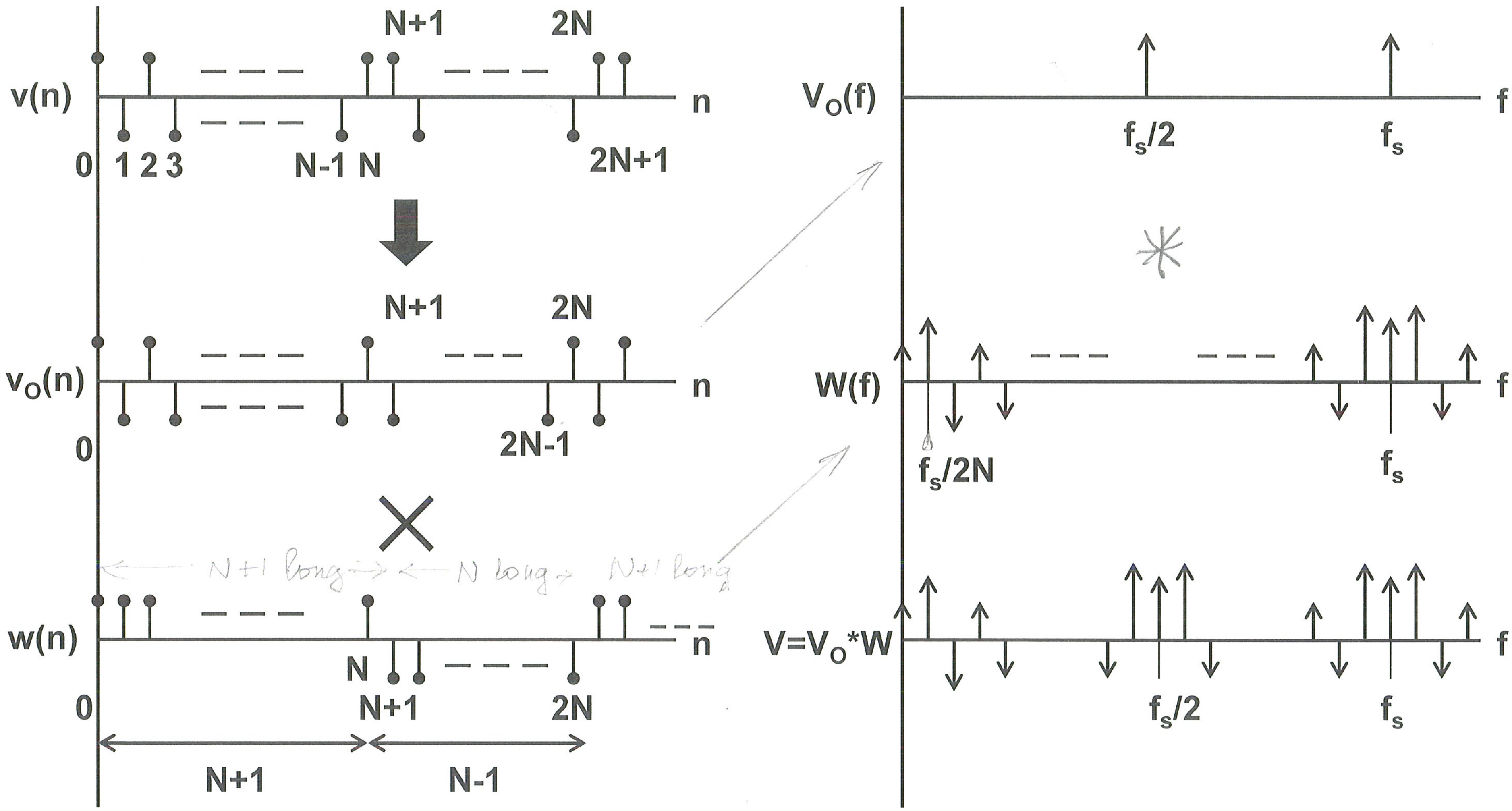
  k = 1, 2, …

$V(f) = \frac{1}{2}\delta(f) + a_1 \delta(f - f_s/4) + \ldots$



$u = 1/500 > 0$

Inband Tone Generation



Oregon State UNIVERSITY OSU

temes@ece.orst.edu

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MOD1 Under DC Excitation (2)

$\times V_{REF}$

- Let $u = a/b$, a and b odd integers, and $0 < a < b$. Also, let $|y(0)| < 1$. Then, the output has a period b samples. In each period, $v(n)$ will contain $(b+a)/2$ samples of +1, and $(b-a)/2$ samples of -1. 1, 1, -1,] 1, 1, -1
- If a or b is even, the period is $2b$, with $(a+b)$ +1s and $(b-a)$ -1s.

See p. 19, $a=1$, $b=2$

- If $v(n)$ has a period p, with m +1s and $(p-m)$ -1s, the average $\overline{v} = (2m - p)/p$. Hence, $u = \overline{v}$ is also rational. Thus, rational dc $u \Leftrightarrow$ periodic $v(n)$.
- Periodic $v(n)$: pattern noise, idle tone, limit cycle. Not instability!
- For $u = 1/100$, tones at $k \cdot f_s/200$, $k = 1, 2, \ldots$ some may be in the baseband. Often intolerable! 99

J. Candy & Benjamin

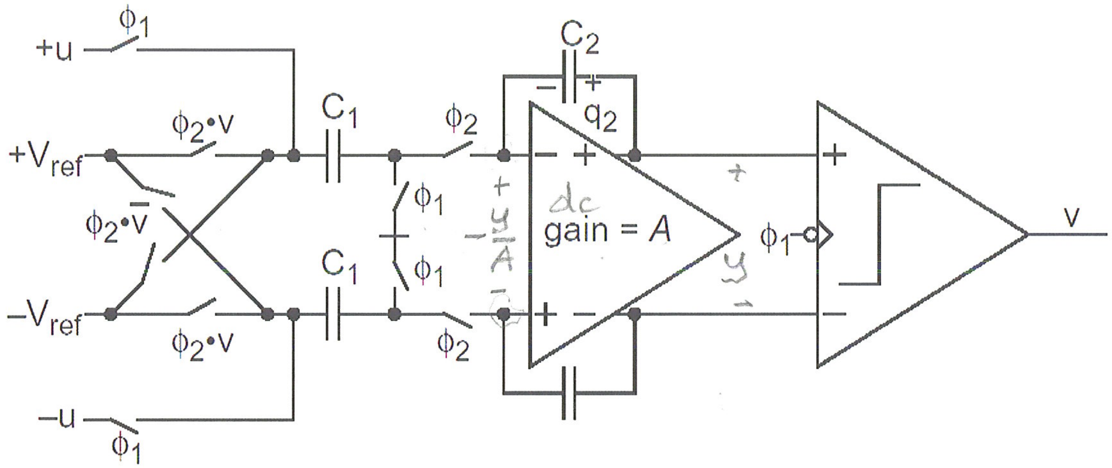
Stability of MOD1

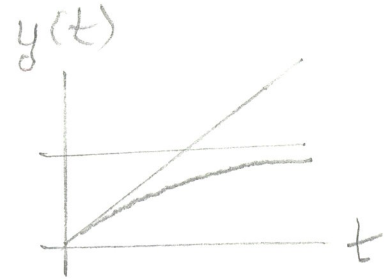
- MOD1 always stable as long as $|u| \le 1$, ~~and $|y(0)| \le 2$:~~

$$y(n) = \underbrace{\left[y(n-1) - \overbrace{\mathrm{sgn}\left(y(n-1)\right)}^{v(n-1)}\right]}_{|[\ ]| \le 1} + \underbrace{u(n)}_{|u| \le 1} \quad \le 2$$

- If $u > 1$ (or $u < -1$), v will always be +1 (or -1) $\Rightarrow$ y will increase (or decrease) indefinitely.
- If $|u(n)| \le 1$ but $|y(0)| > 2$, then $|y(n)|$ will decrease to < 2. Output spectrum always a line spectrum for MOD1 with dc input (rational or not).

The Effects of Finite Op-Amp Gain (1)



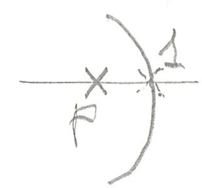


- Degraded noise shaping:

/ error

$$q_2(n) = q_2(n-1) + C_1\left(u(n) - v(n-1) - \frac{q_2(n)}{C_2(A+1)}\right)$$

$V_{ref} = 1$



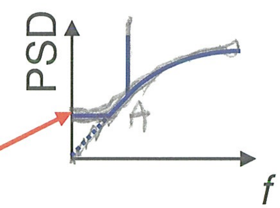
pole: $p = 1 - 1/A$

$$Y(z) = p\frac{zU(z) - V(z)}{z - p}$$

, $V = Y + E$

$$NTF(z) = 1 - pz^{-1} \rightarrow 1 - p = 1/A$$

pole error, dc gain of NTF



The Effects of Finite Op-Amp Gain (2)

- Dead zones: $0 < u \ll 1$ dc

Ideally: $y(1) = y(0) + u - \mathrm{sgn}(y(0)) = u - 1 < 0$ $\quad v =$ -1

$A \to \infty$ $\quad y(2) = (u-1) + u + 1 = 2u > 0$ $\quad$ +1

$y(3) = 2u + u - 1 = 3u - 1 < 0$ $\quad$ -1

$$y(k) = \begin{cases} ku - 1 & \text{, if } k \text{ is odd} \\ ku \; >0 & \text{, if } k \text{ is even} \end{cases}$$

for $u > 0$, eventually $ku > 1$ and two 1's occur.

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For A < ∞: $y(n) = p\,y(n-1) + u - \mathrm{sgn}(y(n-1))$, $p = 1 - 1/A < 1$

$y(1) = y(0) + u - \mathrm{sgn}(y(0)) = u - 1 < 0$ $\quad v =$ -1

$y(2) = pu - p + u + 1 = (1+p)u + (1-p) > 0$ $\quad$ +1

$y(3) = p(1+p)u + p(1-p) + u - 1 = (1 + p + p^2)u - (1 - p + p^2) < 0$ $\quad$ -1

. . .

$$y(k) = \sum_{i=0}^{k-1} p^i u \overset{}{\underset{-}{+}} (-1)^k \sum_{i=0}^{k-1} (-p)^i$$

For odd k & ? $k \to \infty$, $y(k) \lesssim 0$

The Effects of Finite Op-Amp Gain (3)

- For $\bar{v} > 0$,

(Two 1's occuring)

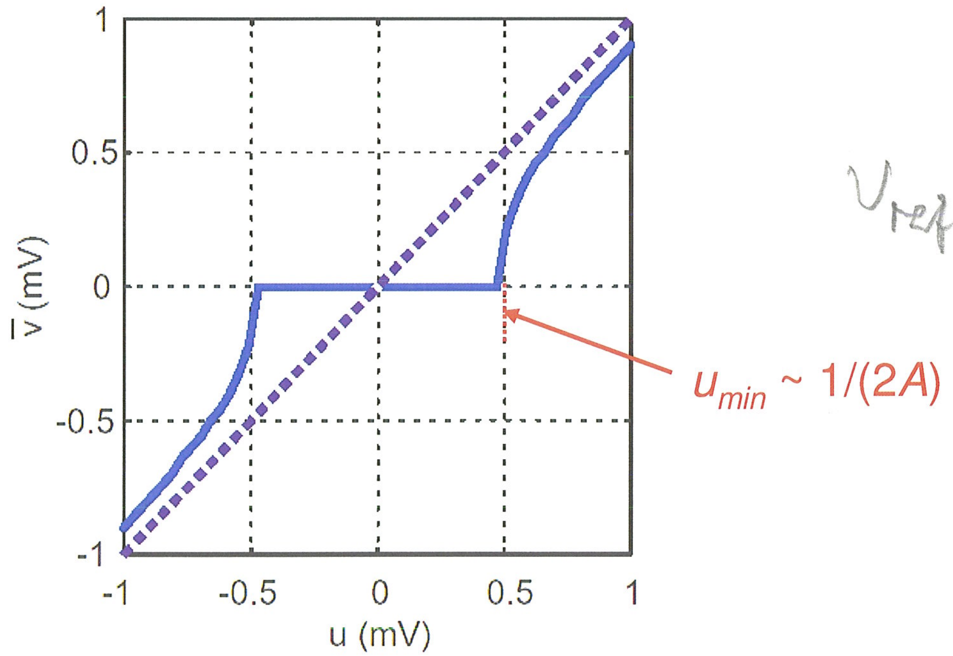
$$\frac{u}{1-p} > \frac{1}{1+p}$$

$$u > \frac{1-p}{1+p} = \frac{1/A}{2-1/A} \approx \frac{1}{2A}$$

$A = 10^3 \sim 10^4$

For $A \approx 10^3$:

Dead zone



$V_{ref} = 1V$

without decimation

With decimation

Noise filtering

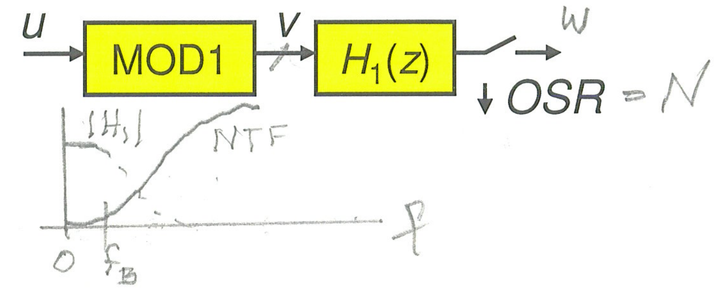
Decimation

Decimation Filters for MOD1 (1)

- The sinc filter:

Averaging over N samples (running-average)

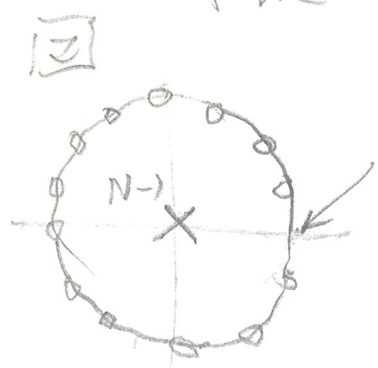
$$w(n) = \frac{1}{N}\sum_{i=0}^{N-1} v(n-i)$$



FIR

$$h_1(n) = \begin{cases} 1/N, & \text{if } (0 \le n \le N-1) \\ 0, & \text{otherwise} \end{cases}$$

FIR



No zero at dc

$$\boxed{H_1(z) = \frac{1}{N}\frac{1-z^{-N}}{1-z^{-1}}} = \frac{1}{N}\left[1 + z^{-1} + \cdots + z^{-N+1}\right]$$

IIR

$z = e^{j\omega T}$ $T = 1$

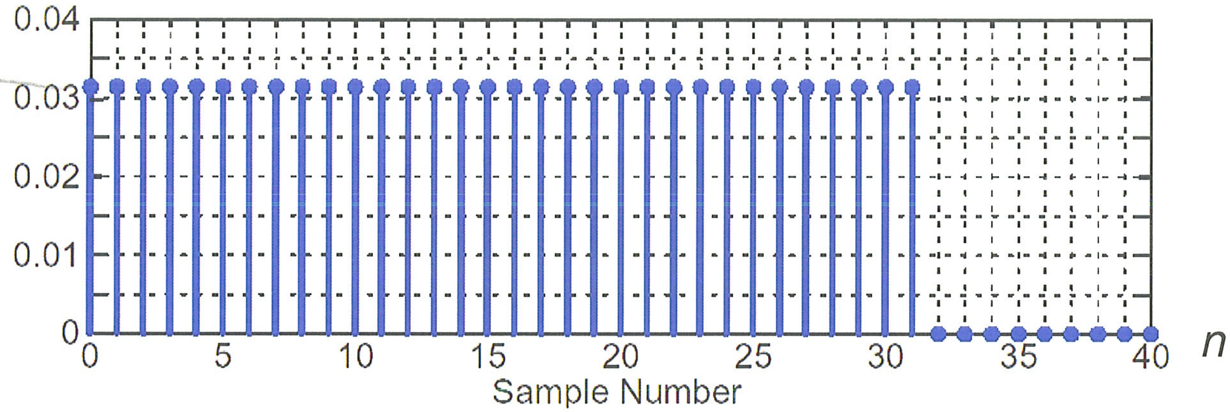
$$H_1(e^{j2\pi f}) = \frac{\operatorname{sinc}(Nf)}{\operatorname{sinc}(f)} \qquad \operatorname{sinc}(f) \triangleq \frac{\sin(\pi f)}{\pi f}$$

Decimation Filters for MOD1 (2)

- Responses:

1/N

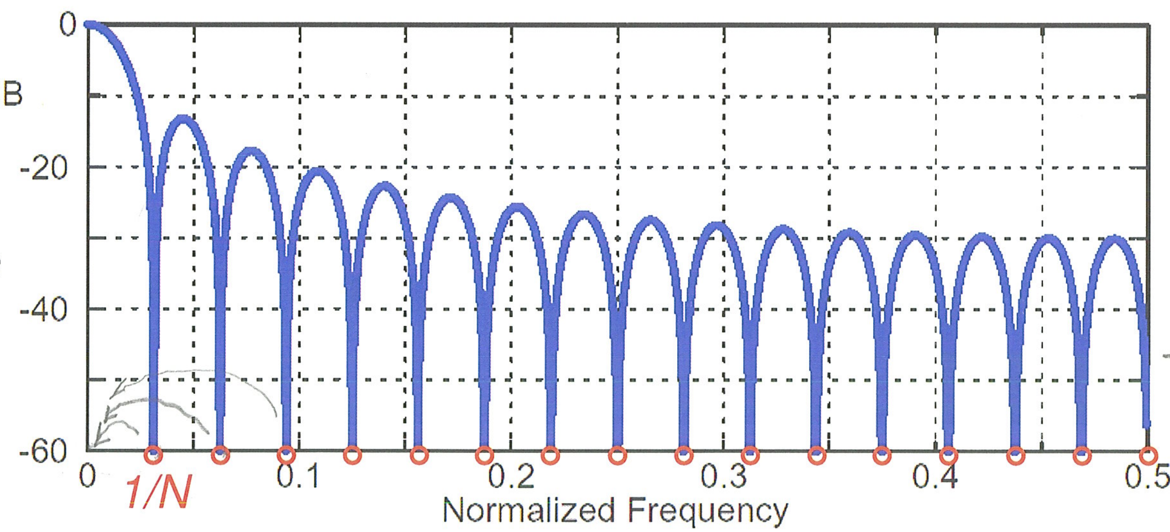
$h_1(n)$



N=32 -(OSR)

OSR = N

Gain response $H_1(z)$



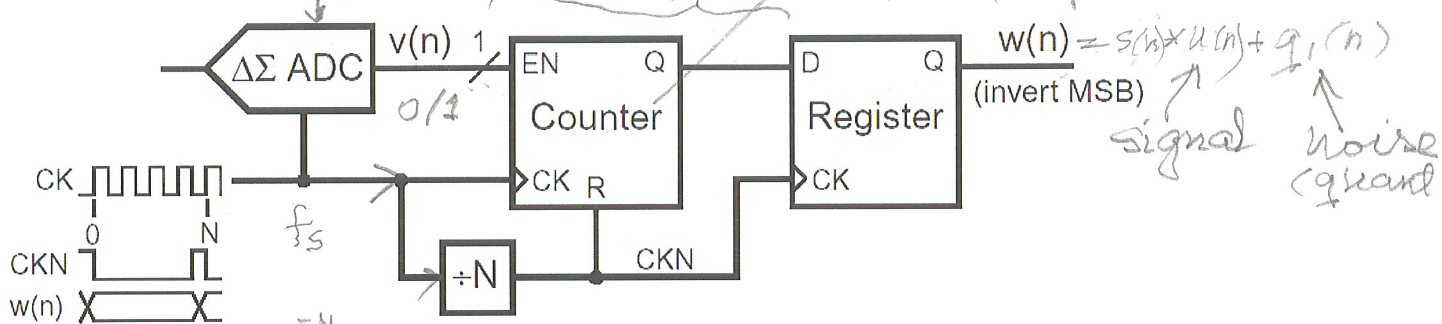
f/f_s

Areas around notches fold back to baseband after decimation if N = OSR.

Decimation Filters for MOD1 (3)

accum./dump
"fill & spill"

- Implementation:

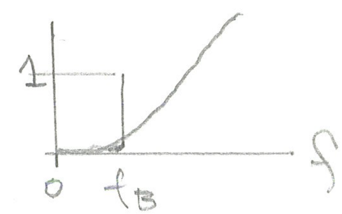


$\frac{1-z^{-N}}{N(1-z^{-1})} \cdot (1-z^{-1})$

$$Q_1(z) = H_1(z)NTF(z)E(z) = \frac{1}{N}(1-z^{-N})E(z)$$

output noise

$$q_1(n) = \frac{1}{N}[e(n) - e(n-N)]$$



Assuming $e(n)$ and $e(n-N)$ are uncorrelated:

ideal LPF

~~Inband~~ noise after H_1:

Total power of

$$\sigma_{q_1}^2 = \frac{2e_{rms}^2}{N^2}$$

σ_e^2

Inband noise before H_1:

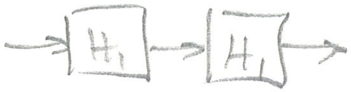
$$\sigma_{q_0}^2 = \frac{\pi^2\sigma_e^2}{3N^3}$$

Total noise after H_1; Too much!

Total noise after ideal LPF; Much less than σ_{q1}^2!

Decimation Filters for MOD1 (4)

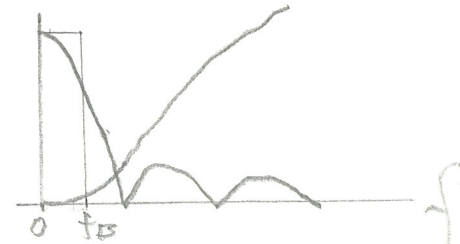
- The sinc² filter:



$$\boxed{H_2(z) = \left[\frac{(1 - z^{-N})}{N(1 - z^{-1})}\right]^2}$$

$$H_2(e^{j2\pi f}) = \left(\frac{\text{sinc}(Nf)}{\text{sinc}(f)}\right)^2$$

$$Q_2(z) = NTF(z)H_2(z)E(z) = \frac{1}{N^2}\frac{(1 - z^{-N})}{(1 - z^{-1})}(1 - z^{-N})E(z) = \frac{1}{N}H_1(z)[(1 - z^{-N})E(z)]$$



$$q_2(n) = \frac{1}{N^2}\sum_{i=0}^{N-1}[e(n-i) - e(n-N-i)]$$

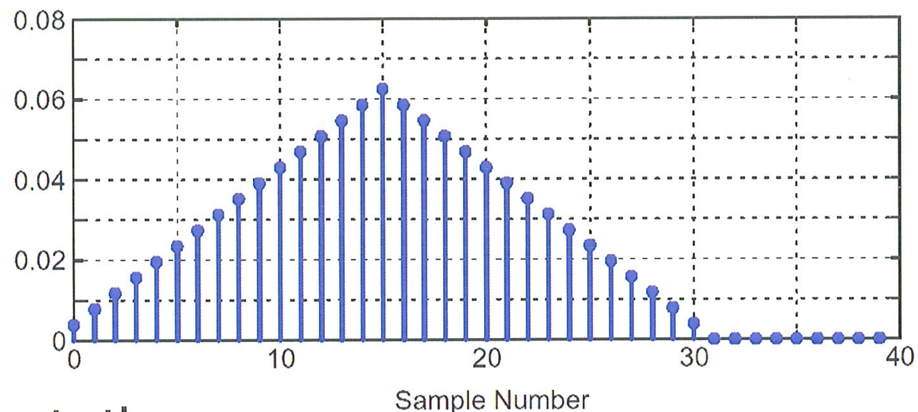
Total power $\sigma_{q_2}^2 = \dfrac{2N\sigma_e^2}{N^4} = \dfrac{2\sigma_e^2}{N^3}$

$\sigma_{q_0}^2 = \dfrac{\pi^2 \sigma_e^2}{3N^3} > \sigma_{q_2}^2$!

(but signal is also reduced by H_2)

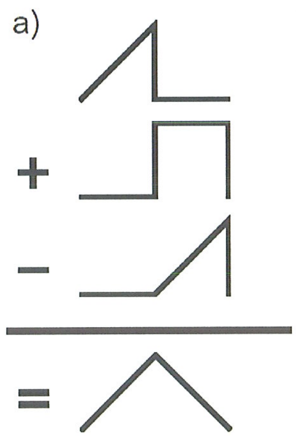
Decimation Filters for MOD1 (5)

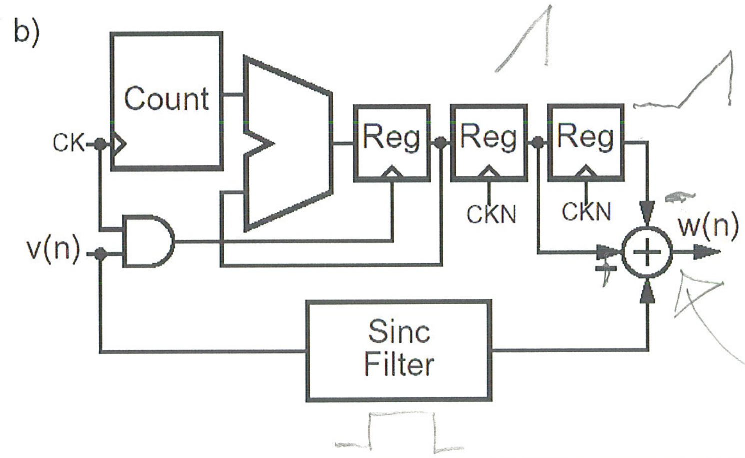
- Response:



$(1 - z^{-1})^L$

- Implementation:





Or Hogenauer

error!

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3. V. Friedman, “The structure of the limit cycles in sigma delta modulation,” *IEEE Transactions on Communications*, vol. 36, no. 8, pp. 972-979, August 1988.
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5. R. M. Gray, “Spectral analysis of quantization noise in a single-loop sigma-delta modulator with dc input,” *IEEE Transactions on Communications,* vol. 37, no. 6, pp. 588-599, June 1989.
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Parameter	Value
input step size (LSB size)	2
output step size	2
number of steps	M $= 2^N - 1$
number of levels	$M + 1$
N = number of bits	$\lceil \log_2(M+1) \rceil$
no-overload input range	$[-(M+1), M+1]$
full-scale	$2M$
input thresholds	$0, \pm 2, \ldots, \pm(M-1)$, M odd $\pm 1, \pm 3, \ldots, \pm(M-1)$, M even
output levels	$\pm 1, \pm 3, \ldots, \pm M$, M odd $0, \pm 2, \pm 4, \ldots, \pm M$, M even

Table 2.1. Properties of the symmetric quantizers of Figs. 2.3 and 2.4.