## CS 271Computer Architecture and Assembly Language

## Self-Check for Lecture\#8

## Solutions

1. 

a. Show the 16 -bit representation of 2437 (decimal).
$2437($ decimal $)=100110000101$ binary $=0000100110000101$ (16-bit)
b. Convert the 16-bit representation of part (a)to the corresponding odd-parity Hamming code. Add the appropriate number of parity bits.

- Since the representation is 16 -bit, we need $\log _{2} 16+1=5$ additional bits, i.e. the Hamming code will have 21 bits.
- Number the places left to right starting with 1 , and put the digits of the 16 -bit number into the places, skipping bits with numbers that are powers of 2:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 0 |  | 0 | 0 | 0 |  | 1 | 0 | 0 | 1 | 1 | 0 | 0 |  | 0 | 0 | 1 | 0 | 1 |

- Bit \#1 determines parity for bits \#1,3,5,7,9,11,13,15,17,19,21
- bits \# $9,13,19$, and 21 are set to 1 , so bit \#1 must be 1 to make odd-parity.
- Bit \#2 determines parity for bits \#2,3,6,7,10,11,14,15,18,19
- bit \#19 is set to 1 , so bit \#2 must be 0 .
- $\quad$ it \#4 determines parity for bits \#4,5,6,7,12, 13, 14, $15,20,21$
- bits \# 12,13 , and 21 are set to 1 , so bit \#4 must be 0 .
- Bit \#8 determines parity for bits \#8,9,10,11, 12,13,14,15
- bits \# 9,12, and 13 are set to 1 , so bit \#8 must be 0 .
- Bit \#16 determines parity for bits \#16,17,18,19,20,21
- bits \# 19 and 21 are set to 1 , so bit \#16 must be 1 .

Answer:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |

2. Given the 21-bit even-parity Hamming code 100001100011100110101.
a. Which bit is incorrect?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
|  |  |  | $t \# 1$ |  | $\begin{aligned} & \text { rmin } \\ & \text { its \# } \\ & \text { le bit } \\ & \text { rmin } \\ & \text { its \# } \\ & 2,3,6 \\ & \text { rmin } \\ & \text { its \# } \\ & \text { Em } \\ & \text { rmin } \\ & \text { Is \# } \\ & \text { rors } \\ & \text { term } \\ & \text { its } \end{aligned}$ | $\begin{aligned} & \text { es pay } \\ & , 7,11 \\ & \text { in } E \\ & \text { es pa } \\ & 7,11, \\ & 7,10, \\ & \text { es pa } \\ & 7,1 \\ & r \text { Set } \\ & \text { es pa } \\ & 1,12, \\ & \text { et8 } \\ & \text { nes p } \\ & 6,17 \end{aligned}$ |  | $\begin{aligned} & \text { r bit } \\ & 7,19, \\ & \text { etl } \\ & \text { r bit } \\ & 19 \text { ar } \\ & 15,1 \\ & \text { r bit } \\ & \text { and } \\ & 5,6, \\ & \text { r bit } \\ & 13 \text { ar } \\ & 10,1 \\ & \text { for bi } \\ & \text { d } 21 \end{aligned}$ | $\begin{aligned} & \# 1,3, \\ & \text { and } \\ & \{1,3, \\ & \# 2,3, \\ & \# \text { set to } \\ & 8,19\} \\ & \# 4,5, \\ & 1 \text { are } \\ & 7,12,13 \\ & \# 8,9, \\ & \text { enet to } \\ & 12,13 \\ & \text { ts } \# 16, \\ & \text { are se } \end{aligned}$ | $\begin{aligned} & 5,7,9 \\ & 1 \text { are } \\ & 5,7,9, \\ & 6,7,1 \\ & 1.7 \\ & 6,7,1 \\ & \text { set to } \\ & 3,14, \\ & 10,11 \\ & 1.7 \\ & 3,14, \\ & 17,1 \\ & 17 \end{aligned}$ |  | $\begin{aligned} & , 15,1 \\ & 1 . \mathrm{T} \\ & 15,1 \\ & 4,15, \\ & \text { four } \\ & 4,15, \\ & \text { at's } \\ & 21\} \text {. } \\ & 3,14, \\ & \text { threa } \\ & 0,21 \\ & \text { t's fo } \end{aligned}$ | 20,2 <br> five <br> 5 <br> 1-b <br> ur 1 | 21 seve $21\}$. <br> , so <br> -bits <br> s, so <br> its, | 1-b <br> he er <br> so th <br> there <br> the | S, SO <br> or is <br> re is <br> is an <br> error | ther <br> not <br> an <br> even | the <br> en-p <br> parity <br> in b | even <br> bits <br> arity <br> err its \#1 | par <br> in <br> 6,17 |

Answer:

- The only number in the intersection of ErrorSet1, ErrorSet4, and ErrorSet8 is 13, so the error is in bit \#13.
- Another way to find it is to add the parity place numbers that have errors: $1+4+8=13$.
b. After the error is corrected, what decimal number is represented by the Hamming code of part (a)?

| - The corrected code is |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 4 | 5 | 6 | 11 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |

- Extract the data bits ... 0011001100010101 , and convert to decimal:

Answer: 13077
3. Note: This is NOT a programming assignment (but you might enjoy programming it anyway).

I need a program to calculate the odds of winning a lottery. The user enters the range of possible numbers and the number of picks required. For example, the user might enter 42 for the range, with 5 picks on one ticket. This will involve calculating the number of combinations of $r$ items taken from a set of $n$ items (i.e., $n C_{r}$ ).

The program should display the odds of winning with one ticket.
For example: The odds of winning with 5 picks from 42 lottery numbers:
1 in 850668
a. How would you modularize this problem?

One way to break the problem into its logical components:
1)display a title screen
2) get the user's numbers
3)calculate the odds
a. calculate nCr
i. calculate factorials
4)display result
b. Show a hierarchy chart of your modularization.


