CS 271
Computer Architecture & Assembly Language

Lecture 16
Reverse Polish Notation (RPN)
Expression Evaluation
IA-32 Floating-Point Unit (FPU)
2/24/22, Thursday

Oregon State University

Odds and Ends

- Final Project posted
    - Due Tuesday, March 15th 11:59 pm

- Due 2/27 11:59 pm:
    - Program 5
    - Weekly Summ~~e~~ary 8
    - Quiz 3

Lecture Topics:

- Reverse Polish Notation (RPN)
- Expression Evaluation
- IA-32 Floating-Point Unit (FPU)

Reverse Polish Notation (RPN) Expression Evaluation

Reverse Polish Notation

- RPN
- Postfix form of expression
- Example
  - Infix: a + (b – c) * (d + e)
  - Postfix (RPN): a bc- de+ * +

    abc-de+*+
- Notice how operator precedence is preserved
- Notice how order of operands is preserved
- Notice how order of operators is **NOT** preserved
- RPN does not require parentheses

Conversion infix ←→postfix (RPN)
Binary Tree Method

- Fully parenthesize infix
- Don’t parenthesize postfix
- **Operands** are always in the original order
- Operators may appear in different order

Conversion infix ←→postfix (RPN)
Binary Tree Method

1. Fully parenthesize the infix expression
   - Follow rules of operator precedence
2. Parse the expression left to right, constructing a binary tree

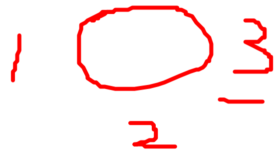
(	go left
Operand	insert
Operator	go up, insert, go right
)	go up

3. Post-order traversal gives RPN

Examples (infix → postfix)

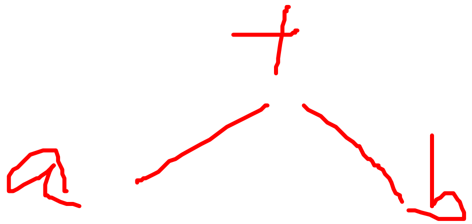
– (	go left
– Operand	insert
– Operator	go up, insert, go right
–)	go up

- (a + b) * (c + d) + e
  - (((a + b) * (c + d)) + e)
  - ab+cd+*e+



- a * b + (c * d) * e
  - ((a * b) + ((c * d) * e))
  - ab*cd*e*+

a b* c d* e * +

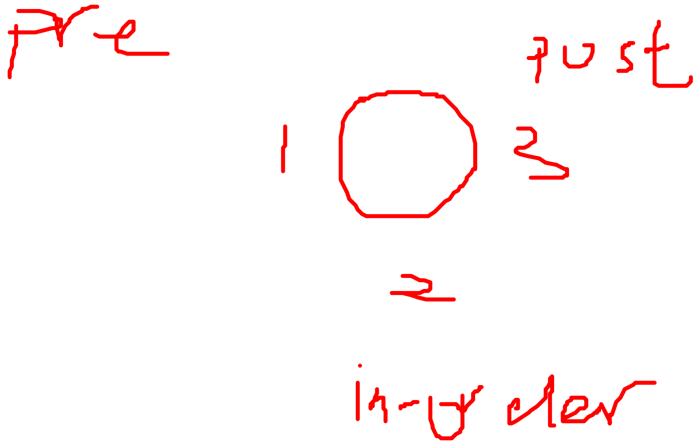


- (a – b) * (((c – d * e) / f) / g) * h
  - ((a – b) * (((c – (d * e)) / f) / g)) * h
  - ab-cde*-f/g/*h*

a b- c d e*-f / g/*h*

Conversion postfix → infix

- Binary tree method
  - Diagram expression as a binary tree
    - Last operator is root
  - Do in-order traversal, parenthesizing each subtree



Example (post → infix)

→

- ab+c+d*
  - ((a + b) + c) * d

((a + b) + c) * d

- abcde+**/
  - a / (b * (c * (d + e)))

a / (b * (c * (d + e)))

- abcde*f/+g-h/*+
  - a + (b * (((c + ((d * e) / f)) – g) / h))

a + (b * (((c + ((d * e) / f)) – g) / h))

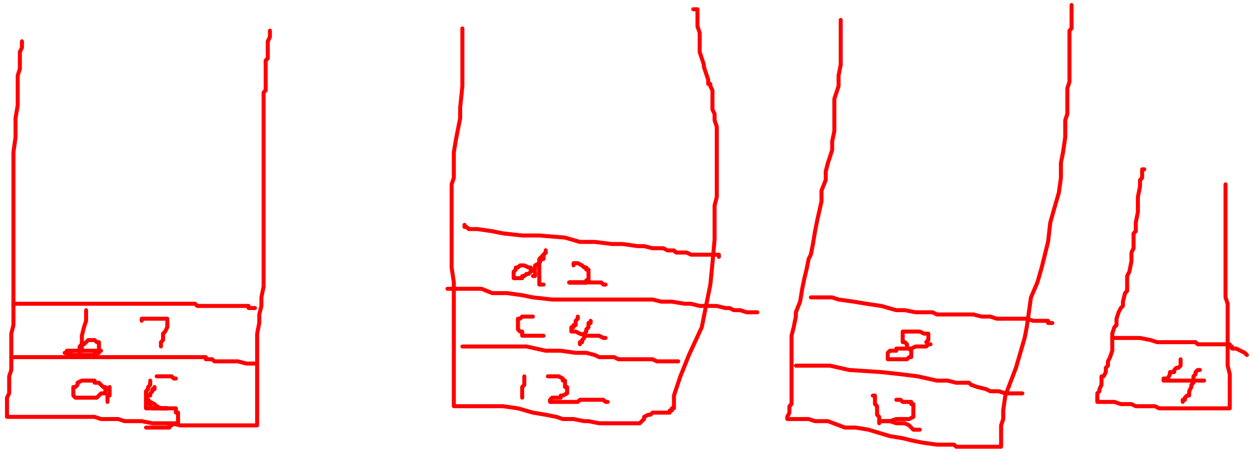
Evaluation of RPN Expressions

- Parse expression left to right, creating a stack of operands
  - Operand: push onto stack
  - Operator: pop 2 operands, perform operation, push result onto stack

- Single value remaining on the stack is value of expression

Examples (Evaluation of RPN Expressions)

- a = 5, b = 7, c = 4, d = 2, e = 3, f = 1, g = 6
- ab+cd*-
  - 4
- abcdefg++**++
  - 92
- abc+de*f/+g-*
  - 55



Using RPN in Programs

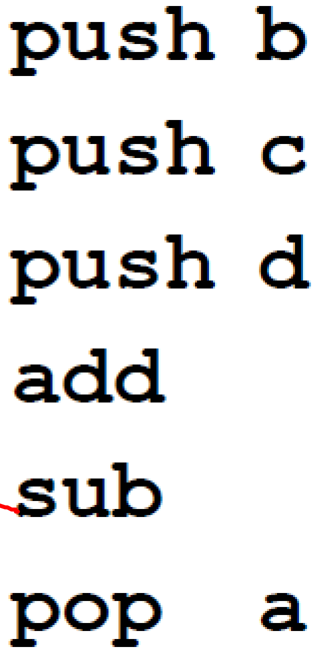
- Expression evaluation
- 0-address machine
  - E.g., Intel IA-32 FPU

- Example: Evaluate **a – b * c**
  1. Convert to RPN abc*-
  2. Program

```
push    a
push    b
push    c
mul
sub
```

a = b – (c + d)

- 0-address: use RPN
- a = b c d + -



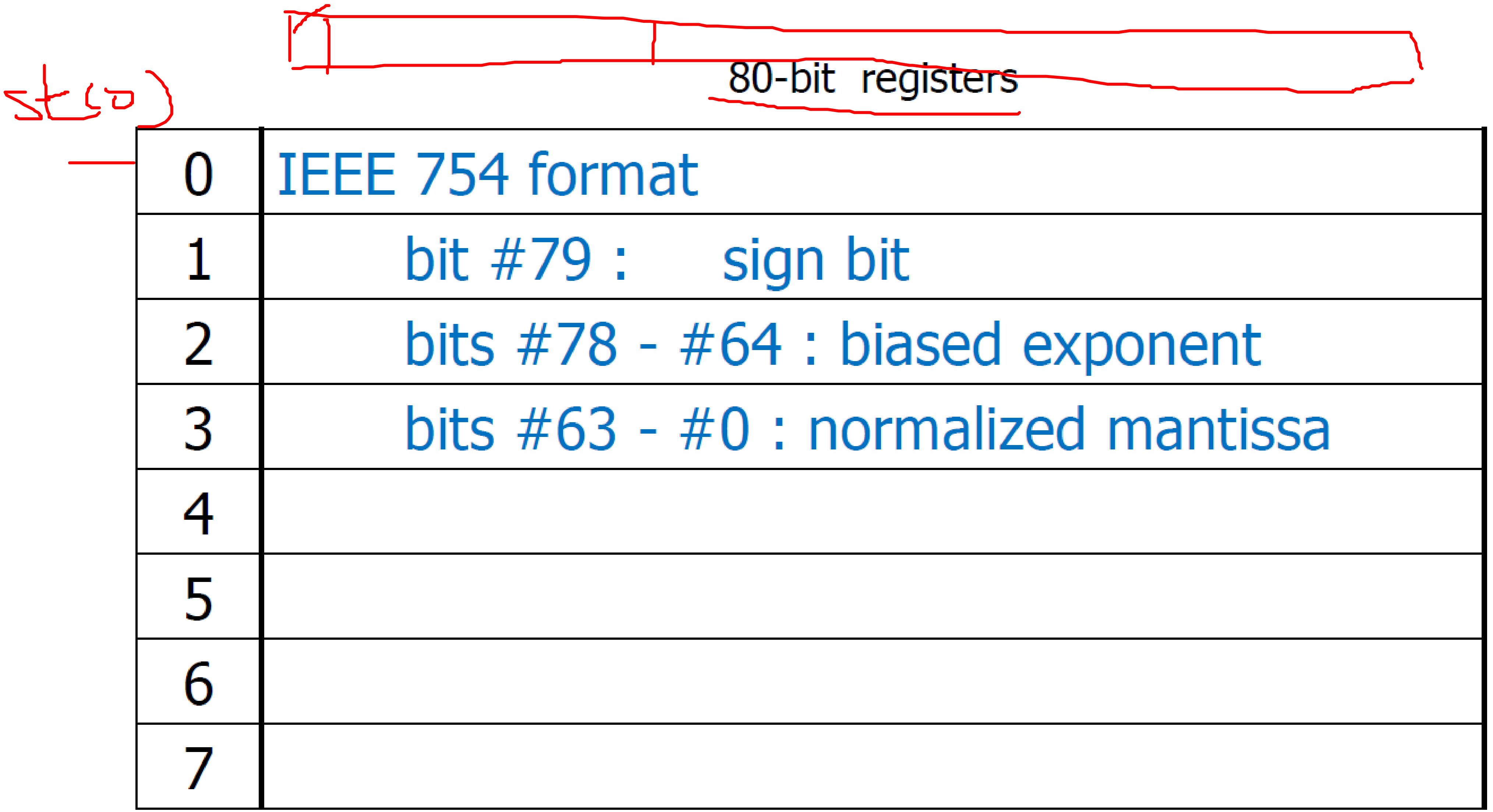
```
push b
push c
push d
add
sub
pop  a
```

IA-32 Floating-Point Unit (FPU)

IA-32 Floating Point Processor (FPU)

- Runs in parallel with integer processor
- Circuits designed for fast computation on floating point numbers (IEEE format)
- Registers implemented as “pushdown” stack
- Usually programmed as a 0-address machine
  - Other instructions are possible
- CPU/FPU Exchange data through memory
  - Converts WORD and DWORD to REAL10

Floating-point Unit Registers



80-bit registers

0	IEEE 754 format
1	bit #79 : sign bit
2	bits #78 - #64 : biased exponent
3	bits #63 - #0 : normalized mantissa
4	
5	
6	
7	

Floating-point Unit Registers

- FPU is programmed as a “pushdown” **stack**
- If you push more than 8 values, the “bottom” of the stack will be lost
- Operations are defined for the “top” one or two registers
- Registers may be referenced by name ST(x)

0 – 7

Programming the FPU

- FPU Registers = ST(0) … ST(7)
- ST=ST(0)=top of the stack
- ST(0) is implied when an operand is not specified
- Instruction Format:

OPCODE

OPCODE destination

OPCODE destination, source

*Restrictions: One register must be ST(0)

- FINIT: initialize FPU register stack
  - Execute before any other FPU instructions!

Programming the FPU

- Note: (FPU instructions begin with ‘F’)
  - FSUB
  - FADD
  - Etc.

- To specify that a value being used is integer, use the special instructions that start with “FI”
  - FISUB
  - FIADD
  - Etc.

Sample Register Stack Opcodes

- FLD MemVar
  - Push ST(i) “down” to ST(i+1) for i = 0…6
  - Load ST(0) with MemVar
- FST MemVar
  - Move top of stack to memory
  - Leave result in ST(0)
- FSTP MemVar
  - Pop top of stack to memory
  - Move ST(i) “up” to ST(i-1) for i = 1…7

Sample FPU Opcodes

- Instructions use top of register stack as implied operand(s)
  - FADD: Addition (pop top two, add, push result)
  - FSUB: Subtraction
  - FMUL: Multiplication
  - FDIV: Division
  - FDIVR: Division (reverses operands)
  - FSIN: Sine (uses radians)
  - FCOS: Cosine (uses radians)
  - FSQRT: Square Root
  - FABS: Absolute Value
  - FYL2X: $Y * \log_2 (X)$ (X is in ST(0), Y is in ST(1)
  - FYL2XP1: $Y * \log_2 (X)+1$

Example

```
.data
varX      REAL10        2.5
varY      REAL10        -1.8
varZ      REAL10        0.9
result    REAL10        ?

.code
     FINIT
     FLD varX
     FLD varY
     FLD varZ
     FMUL
     FADD
     FSTP result      ; result = varX + (varY * varZ)

     ; etc.
```

Example

$\vec{w}$ $\vec{v}$ $\vec{w} \cdot \vec{v}$ (

(6, 4.5) (2, 3.2)

```
; Implementation of (6.0 * 2.0) + (4.5 * 3.2)
; Note: RPN is    6.0  2.0  *  4.5  3.2  *  +

.data
array       REAL10 6.0, 2.0, 4.5, 3.2
dotProduct REAL10 ?
```

0-Address Operations

```
        array       REAL10 6.0, 2.0, 4.5, 3.2

    main PROC    ; RPN is   6.0  2.0  *  4.5  3.2  *  +
      finit              ; initialize FPU
      fld  array         ; push 6.0 onto the stack
      fld  array+10      ; push 2.0 onto the stack
      fmul               ; ST(0) = 6.0 * 2.0
      fld  array+20      ; push 4.5 onto the stack
      fld  array+30      ; push 3.2 onto the stack
      fmul               ; ST(0) = 4.5 * 3.2
      fadd               ; ST(0) = ST(0) + ST(1)
      fstp dotProduct          ; pop stack into memory
      exit
    main ENDP
    END main
```

Irvine’s Library

- ReadFloat: get keyboard input into ST(0)
- WriteFloat: display ST(0) contents in floating-point format

- Experiment!!