Noise in Analog CMOS ICs

Gábor C. Temes

School of Electrical Engineering and Computer Science
Oregon State University
Noise

• Intrinsic (inherent) noise:
  – generated by random physical effects in the devices.

• Interference (environmental) noise:
  – coupled from outside into the circuit considered.

• Switching noise:
  – charge injection, clock feedthrough, digital noise.

• Mismatch effects:
  – offset, gain, nonuniformity, ADC/DAC nonlinearity errors.

• Quantization (truncation) “noise”:
  – in internal ADCs, DSP operations.
Topics Covered

- Types of Noise in Analog Integrated Circuits
- The characterization of Continuous and Sampled Noise
- Thermal Noise in OpAmps
- Thermal Noise in Feedback Amplifiers
- Noise in an SC Branch
- Noise Calculation in Simple SC Stages
- Sampled Noise in OpAmps
Characterization of Continuous-Time Random Noise – (1)

- Noise $x(t)$ – e.g., voltage. Must be stationary as well as mean and variance ergodic.
- Average power defined as mean square value of $x(t)$:
  \[ P_{av} = \lim_{T \to \infty} \left\{ \frac{1}{T} \int_{0}^{T} x^2(t) dt \right\} = E\left\{ x^2(t) \right\} \]
- For uncorrelated zero-mean noises
  \[ E\left\{ (x_1 + x_2)^2 \right\} = E\{x_1^2\} + E\{x_2^2\} \]
- Power spectral density $S_x(f)$ of $x(t)$: $P_{av}$ contained in a 1Hz BW at $f$.
  Measured in V$^2$/Hz. Even function of $f$.
- Filtered noise: if the filter has a voltage transfer function $H(s)$, the output noise PSD is $|H(j2\pi f)|^2 |S_x(f)|$. 
Characterization of Continuous-Time Random Noise – (2)

• Average power in $f_1 < f < f_2$:

$$P_{f_1,f_2} = \int_{f_1}^{f_2} S_x(f) df$$

if $S_x$ is regarded as a one-sided PSD.

Hence, $P_{0,\infty} = P_{av}$. White noise has infinite power!?

• Amplitude distribution: probability density function (PSD) $p_x(x)$. $p_x(x_1)$ $\Delta x$: probability of $x_1 < x < x_1 + \Delta x$ occurring. E.g., $p_x(q) = 1/\text{LSB}$ for quantization noise $q(t)$ if $|q| < \text{LSB}/2$, 0 otherwise.
Characterization of Sampled-Data Random Noise

- Noise $x(n)$ – e.g., voltage samples.
- Ave. power:
  $$P_{av} = E \left[ \lim_{N \to \infty} \left\{ \frac{1}{N+1} \sum_{n=0}^{N} |x(n)|^2 \right\} \right]$$

  $P_{av}$ is the mean square value of $x(n)$.
- For sampled noise, $P_{av}$ remains invariant!
- Autocorrelation sequence of $x(n)$
  $$r_x(k) = E\{x(n) \cdot x(n-k)\}$$
  $$r_x(0) = \left[ P_{av} \text{ of } x(n) \right] = E\{|x(n)|^2\}$$
- PSD of $x(n)$: $S_x(f) = \mathcal{F}[r_x(k)]$. Periodic even function of $f$ with a period $f_c$. Real-valued, non-negative. White noise has finite power!
- Power in $f_1 < f < f_2$: $P_{f_1, f_2} = \int_{f_1}^{f_2} S_x(f) df$
  $$P_{av} = \int_{0}^{f_{c/2}} S_x(f) df = P_{0, f_{c/2}} = E\{|x(n)|^2\}$$
  White noise has finite power!
- PDF of $x(n)$: $p_x(x)$, defined as before.
Thermal Noise – (1)

Due to the random motion of carriers with the MS velocity $\propto T$. Dominates over shot noise for high carrier density but low drift velocity, occurring, e.g., in a MOSFET channel.

Mean value of velocity, $V, I$ is 0.

The power spectral density of thermal noise is $PSD = kT$. In a resistive voltage source the maximum available noise power is hence

$$\frac{E^2}{4 \cdot R} = k \cdot T \cdot BW$$

giving

$$E^2 = 4 \cdot k \cdot T \cdot BW \cdot R$$
Thermal Noise – (2)

The probability density function of the noise amplitude follows a Gaussian distribution

\[ p(E) = \frac{e^{-E^2/\overline{E}^2}}{\sqrt{\overline{E}^2 \cdot 2\pi}} \]

Here, \( \overline{E}^2 \) is the MS value of \( E \).

In a MOSFET, if it operates in the triode region, \( R=r \) can be used.

In the active region, averaging over the tapered channel, \( R=(3/2)/g_m \) results. The equivalent circuit is

\[ \overline{E^2(f)} = \frac{8kT}{g_m} (PSD) \]
Noise Bandwidth

Let a white noise $x(t)$ with a PSD $S_x$ enter an LPF with a transfer function:

$$H(s) = \frac{G_0}{s/\omega_{3\text{-dB}} + 1}$$

where $G_0$ is the dc gain, and $\omega_{3\text{-dB}}$ is the 3-dB BW of the filter. The MS value of the output noise will be the integral of $|H|^2 \cdot S_x$, which gives

$$\overline{x^2} = \frac{\omega_{3\text{-dB}}}{4} G_0^2 \cdot S_x$$

Assume now that $x(t)$ is entered into an ideal LPF with the gain function:

$$|H| = G_0 \quad \text{if} \quad f < f_n$$

and $0$ if $f > f_n$. The MS value of the output will then be:

$$\overline{x^2} = f_n \cdot G_0^2 \cdot S_x$$

Equating the RHSs reveals that the two filter will have equivalent noise transfer properties if

$$f_n = \frac{\omega_{3\text{-dB}}}{4} = \frac{\pi}{2} f_{3\text{-dB}}$$

$f_n$ is the noise bandwidth of the LPF.
Analysis of 1/f Noise in Switched MOSFET Circuits

Hui Tian and Abbas El Gamal, Fellow, IEEE

Abstract—Analysis of 1/f noise in MOSFET circuits is typically performed in the frequency domain using the standard stationary 1/f noise model. Recent experimental results, however, have shown that the estimates using this model can be quite inaccurate especially for switched circuits. In the case of a periodically switched transistor, measured 1/f noise power spectral density (psd) was shown to be significantly lower than the estimate using the standard 1/f noise model. For a ring oscillator, measured 1/f-induced phase noise was shown to be significantly lower than the estimate using the standard 1/f noise model. For a source follower reset circuit, measured 1/f noise power was also shown to be lower than the estimate using the standard 1/f model. In analyzing noise in the follower reset circuit using frequency-domain analysis, a lower cutoff frequency that is inversely proportional to the circuit on-time is assumed. The choice of this lower cutoff frequency is quite arbitrary and can cause significant inaccuracy in estimating noise power. Moreover, during reset, the circuit is not in steady state, and thus frequency-domain analysis does not apply.

This paper proposes a nonstationary calculation of the standard 1/f noise model, which allows us to analyze 1/f noise in switched MOSFET circuits more accurately. Using our model, we analyze noise for the three aforementioned switched circuit examples and obtain results that are consistent with the reported measurements.

Index Terms—1/f noise, CMOS image sensor, nonstationary noise model, periodically switched circuits, phase noise, ring oscillator, time-domain noise analysis.

especially for switched circuits. An important class of such circuits is periodically switched circuits, which are widely used in RF applications, such as switched capacitor networks, modulators and demodulators, and frequency converters. In the simplest case of a periodically switched transistor, it was shown that the measured drain voltage 1/f noise power spectral density (psd) [5]-[7] is much lower than the estimate using the standard 1/f noise model. Another example that has recently been receiving much attention is 1/f-induced phase noise in CMOS oscillators [8]-[16]. Unlike the amplitude fluctuations, which can be practically eliminated by applying limiters to the output signal, phase noise cannot be reduced in the same manner. As a result, phase noise limits the available channels in wireless communications. Recent measurements [7] show that the 1/f-induced phase noise psd in ring oscillators is much lower than the estimate using the standard 1/f noise model.

Yet another example of a switched circuit is the source follower reset circuit, which is often used in the output stage of a charge-coupled device (CCD) image sensor [11] and the pixel circuit of a CMOS active pixel sensor (APS) [12]. To find the output noise power due to 1/f noise, frequency-domain analysis is typically performed using the stationary 1/f noise model. A low cutoff frequency f_c that is inversely proportional to the

- 1/f noise can be represented as threshold voltage variation.
- If the switch is part of an SC branch, it is unimportant.
- In a chopper or modulator circuit, it may be very important. See the TCAS paper shown.
Thermal Op-Amp Noise – (1)

Simple op-amp input stage [3]:

With device noise PSD: \( V_{n}^2 = \frac{8}{3} kT / g_{m} \)

Equivalent input noise PSD: \( V_{neq}^2(f) = \frac{16}{3} \frac{kT}{g_{m1}} \left[ 1 + \frac{g_{m3}}{g_{m1}} \right] \approx \frac{16}{3} \frac{kT}{g_{m1}} V^2 / Hz \)

For \( g_{m1} \gg g_{m3} \). Hence, it can be represented by a noisy resistor \( R_N = \frac{(8/3)kT}{g_{m1}} \) at one input terminal. Choose \( g_{m1} \) as large as practical!
Thermal Op-Amp Noise – (2)

All devices in active region, \([id(f)]^2 = (8/3)kT \cdot g_m\). Consider the short-circuit output current \(I_{o,sh}\) of the opamp. The output voltage is \(I_{o,sh} \cdot R_o\).

If the \(i\)th device PSD is considered, its contribution to the PSD of \(I_{o,sh}\) is proportional to \(g_{mi}\). Referring it to the input voltage, it needs to be divided by the square of the input device \(g_m\), i.e., by \(g_m^2\).

Hence, the input-referred noise PSD is proportional to \(g_m/g_{m1}^2\). For the noise of the input device, this factor becomes \(1/g_{m1}\).

Conclusions: Choose \(1/g_{m1}\) as large as possible. For all noninput devices (loads, current sources, current mirrors, cascade devices) choose \(1/g_{m3}\) as small as possible!
Noisy Op-Amp in Unity-Gain Feedback

Consider an op-amp with thermal input noise PSD \( P_{ni} = 16kT/3g_{m1} \), where \( g_{m1} \) is the transconductance of the input devices. In a unity-gain feedback configuration:

\[
\begin{align*}
V_{in} & \quad \bigcirc \quad V_n \quad \bigcirc \quad V_o \\
\end{align*}
\]

We shall assume a single-pole model for the op-amp, with a voltage gain \( A(s) = A_0 \omega_{3\text{-dB}}/(s+\omega_{3\text{-dB}}) \), where \( A_0 \) is the DC gain, \( \omega_{3\text{-dB}} \) is the 3-dB BW (pole frequency), and \( \omega_u = A_0 \omega_{3\text{-dB}} \) is the unity-gain BW of the op-amp. For folded-cascode telescopic and 2-stage OTAs, usually \( \omega_u = g_{m1}/C \), where \( C \) is the compensation capacitor and \( g_{m1} \) is again the transconductance of the input devices. Then the open-loop noise BW of the Op-Amp is \( f_n = g_{m1}/4A_0C \), and the open-loop noise gain at DC is \( A_0 \). Hence, the open-loop output noise power is

\[
P_{on} = 16 \frac{kT}{3g_{m1}} A_0^2 \frac{g_{m1}}{4A_0C} = \frac{4A_0kT}{3C}.
\]

If the op-amp is in a unity-gain configuration, then (for \( A_0 \gg 1 \)) the noise bandwidth of the stage becomes \( A_0f_n \), and the DC noise gain is 1. Hence, the output (and input) thermal noise power is

\[
P_n = 16kT \frac{g_{m1}}{3g_{m1}} = \frac{4kT}{3C},
\]

This result is very similar to the \( kT/C \) noise power formula of the simple R-C circuit.
Noisy Op-Amp in a Gain Stage

A more general feedback stage is shown below:

Let the ideal stage gain $G_i = Y_1/Y_2$ be constant. Then the noise voltage gain is the single-pole function $A_n(s) = V_{on}(s)/V_n(s) = \omega_u/(s + \omega_{3-dB}')$ where $\omega_{3-dB}' = \omega_u/(1 + G_i)$ is the 3-dB frequency of $A_n(j\omega)$. The DC noise gain is $\omega_u / \omega_{3-dB}' = 1 + G_i$, and the noise BW of the stage $f_n' = g_{m1}/[4C(1 + G_i)]$ is $1 + G_i$. Hence, the output thermal noise power is

$$P_{no} = \frac{16kT}{3g_{m1}} (1 + G_i)^2 \frac{g_{m1}/4C}{1 + G_i} = \frac{4(1 + G_i)kT}{3C},$$

and the input-referred thermal noise power is

$$P_{ni} = \frac{4}{3} \frac{1 + G_i kT}{G_i^2} C.$$

Note that $P_{ni}$ is smaller for a higher gain $G_i$, so a higher SNR is possible for higher stage gains.
Switched-Capacitor Noise – (1)

Two situations; example:

Situation 1: only the sampled values of the output waveform matter; the output spectrum may be limited by the DSP, and hence $V_{RMS,n}$ reduced. Find $V_{RMS}$ from $\sqrt{kTC}$ charges; adjust for DSP effects.

Situation 2: the complete output waveform affects the SNR, including the S/H and direct noise components. Usually the S/H dominates. Reduced by the reconstruction filter.
Switched-Capacitor Noise – (2)

\[ V_{in} = 0 \]

\[ \phi_1 \]

\[ C \]

\[ \phi_1 \]

\[ R \] (noisy)

\[ C \]

\[ \phi_1 \]

\[ R \] (ideal)

\[ V_n \]

\[ + \]

\[ V_{cn} \]
Switched-Capacitor Noise – (3)

Thermal noise in a switched-capacitor branch: (a) circuit diagram; (b) clock signal; (c) output noise; (d) direct noise component; (e) sampled-and-held noise component. The noise power is $kT/C$ in every time segment.
Switched-Capacitor Noise Spectra (one-sided) - (4)

For $f \ll f_c$, $S_{S/H} \gg S^D$!
Switched-Capacitor Noise – (5)

The MS value of samples in $V_{cn}^{S/H}$ is unchanged:

$$\left(V_{cn}^{S/H}\right) = kT / C$$

Regarding it as a continuous-time signal, at low frequency its one-sided PSD is

$$S^{S/H}(f) \equiv \frac{2(1-m)^2 kT}{f_c C}$$

while that of the direct noise is

$$S^d(f) \equiv \frac{mkT}{f_{sw} C},$$

$$\frac{S^{S/H}}{S^d} = \frac{2(1-m)^2}{m} \frac{f_{sw}}{f_c}.$$ 

Since we must have $f_{sw}/f_c > 2/m$, usually $|S^{S/H}| >> |S^d|$ at low frequencies. (See also the waveform and spectra.)

In the switch-capacitor branch, when the switch is on, the capacitor charge noise is lowpass-filtered by $R_{on}$ and $C$. The resulting charge noise power in $C$ is $kTC$. It is a colored noise, with a noise-bandwidth $f_n = 1/(4 \cdot R_{on} \cdot C)$. The low-frequency PSD is $4kTR_{on}$.

When the switch operates at a rate $f_c << f_n$, the samples of the charge noise still have the same power $kTC$, but the spectrum is now white, with a $PSD = 2kTC/f_c$. For the situation when only discrete samples of the signal and noise are used, this is all that we need to know.

For continuous-time analysis, we need to find the powers and spectra of the direct and S/H components when the switch is active. The direct noise is obtained by windowing the filtered charge noise stored in $C$ with a periodic window containing unit pulses of length $m/f_c$. This operation (to a good approximation) simply scales the PSD, and hence the noise power, by $m$. The low-frequency PSD is thus $4mkTR_{on}$.
To find the PSD of the S/H noise, let the noise charge in C be sampled-and-held at fc, and then windowed by a rectangular periodic window

\[ w(t) = 0 \quad \text{for } n/f_c < t < n/f_c + m/f_c \]
\[ w(t) = 1 \quad \text{for } n/f_c + m/f_c < t < (n+1)/f_c \]
\[ n = 0, 1, 2, \ldots \]

Note that this windowing reduces the noise power by \((1 - m)^2\), since the S/H noise is not random within each period.

Usually, at low frequencies the S/H noise dominates, since it has approximately the same average power as the direct noise, but its PSD spectrum is concentrated at low frequencies. As a first estimate, its PSD can be estimated at \(2(1-m)^2kT/f_c\cdot C\) for frequencies up to \(f_c/2\).
Circuit Example 1: ΔΣ ADC Input Stage

Two sampling operations result in $q_2$ having a $q_{2,n} = \sqrt{2kTC_1}$ RMS noise component. Hence, the input-referred noise of the SC stage is $V_{in,n}^{SC} = \sqrt{2kT/C_1}$. The one-sided $PSD=4kT/f_cC_1$. Including the LPF, the input-referred RMS noise voltage in the baseband becomes:

$$V_{in,n} = \sqrt{2 \times \frac{2kT}{f_cC_1} f_0} = \sqrt{\frac{2kT/C_1}{OSR}}$$

Where $OSR \triangleq f_c/2f_0$. Independent of $R_{on}$; may be used to set $C_{1\text{min}}, OSR_{\text{min}}, T_{\text{max}}$. 

$$q_2 = C_1[v_{D/A}(n) + v_{n2}(n) - v_{n4}(n) - v_{n2}(n-1/2) - v_{n1}(n-1/2) - v_{n3}(n-1/2)]$$
RMS noise charge delivered into \( C_3 \) as \( \phi_2 \rightarrow 0 \), assuming OTA:

From \( C_1 \):
\[
q_1 = \sqrt{2kTc_1} \frac{C_3}{C_2 + C_3}
\]

Form \( C_2 \):
\[
q_2 = \left[ (kTC_2) \left( \frac{C_3}{C_2 + C_3} \right)^2 + kT \frac{C_2C_1}{C_2 + C_3} \right]^{1/2}
\]

Total:
\[
q_3 = \frac{C_3}{C_2 + C_3} \sqrt{kT} \left[ 2C_1 + 2C_2 + \frac{C_2^2}{C_3} \right]^{1/2}
\]

Input-referred RMS noise voltage:
\[
V_{in,n} = q_3 \frac{C_2 + C_1}{C_1C_3} = \frac{\sqrt{kT}}{C_1} \left[ 2C_1 + 2C_2 + \frac{C_2^2}{C_3} \right]^{1/2}
\]

\[
V_{in,n} = \frac{1}{C_1} \sqrt{kT(C_1 + C_2)} \quad \text{for} \quad C_2 \ll C_3.
\]

\( V_{in,n} \) and \( V_{in} \) are both low-pass filtered by the stage.
Sampled Op-Amp Noise Example [4]

• $\phi_1 = 1$
  Direct noise output voltage = $V_{neq}$

• $\phi_2 = 1$
  Charges delivered by $C_1$ and $C_2$: 
  $$-C_1(V_{neq} + V_{in}) + C_2(V_0 - V_{neq})$$
  Charge error $-(C_1 + C_2)V_{neq}$.

• Input-referred error voltage
  $$V_{neq}(1 + C_2/C_1)$$
Reference


