

CS 162

Intro to Computer Science II

Lecture 21

Recursion

3/6/24



Oregon State
University

Today's topic(s)

- Recursion

fun A() :

fun A() :

fun B() :

fun B()

fun A()

Recursion

- Function that calls itself 1 or more times (directly or indirectly)
- Form of repetition
- Has one or more base case for stopping
- Inductive reasoning: general case must eventually be reduced to a base case

Example: Factorial

- Definition

$$\underline{n!} = n * (n-1) * \dots * (n-(n-1)) * 1 = n * (n-1)! ; n > 0$$

Base case: $0! = 1$;

$$n! = n * \frac{(n-1)!}{1} \\ (n-1) * (n-2) !$$

Iterative Factorial

```
int factorial(int n) {  
    int fact;  
    if (n==0)  
        fact=1;  
    else  
        for (fact=n; n > 1; n--)  
            fact = fact*(n-1);  
    return fact;  
}
```

Computing Factorial Iteratively

factorial(4)

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\text{factorial}(4) = 4 * 3$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```


Computing Factorial Iteratively

$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \\ &= 24 * 1\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \\ &= 24 * 1 \\ &= 24\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Recursive Factorial

```
int factorial(int n) {  
    if (n == 0)          // Base case  
        return 1;  
    else  
        return n * factorial(n - 1);    // Recursive call  
}
```

Computing Factorial Recursively

factorial(4)

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\text{factorial}(4) = 4 * \text{factorial}(3)$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1)))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0))))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1)))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2)\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

$$\begin{aligned}\underline{\text{factorial}(4)} &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \\ &= 24\end{aligned}$$

Recursive Factorial

main):
fact (4)

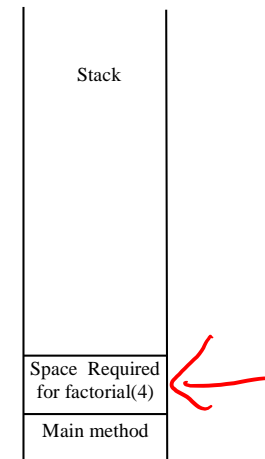
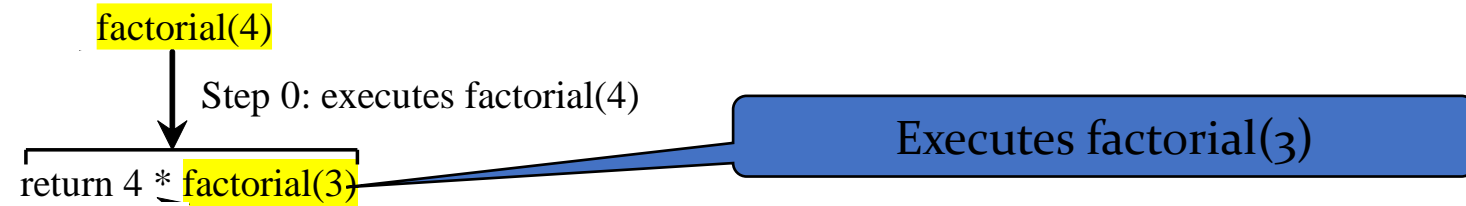
factorial(4)

Executes factorial(4)

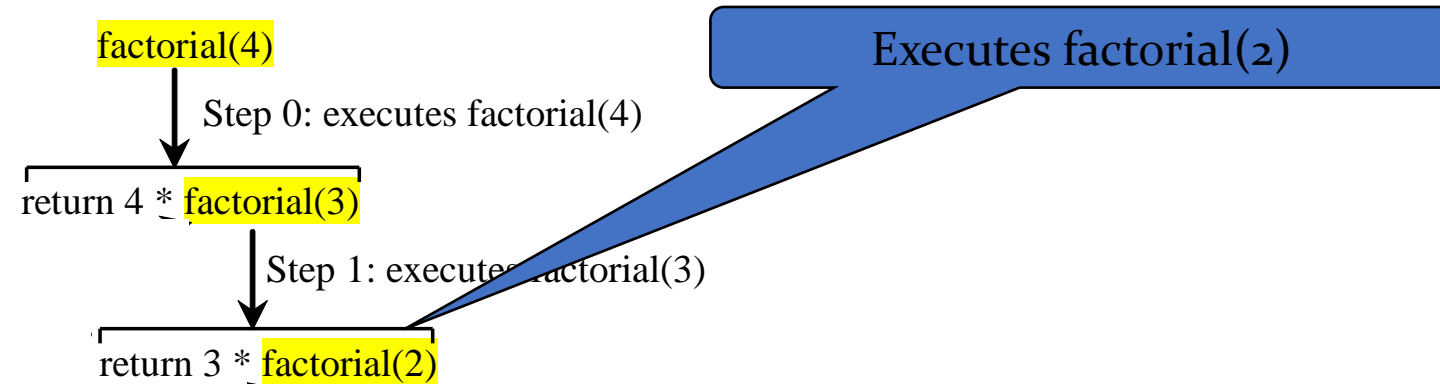
Stack

Main method

Recursive Factorial

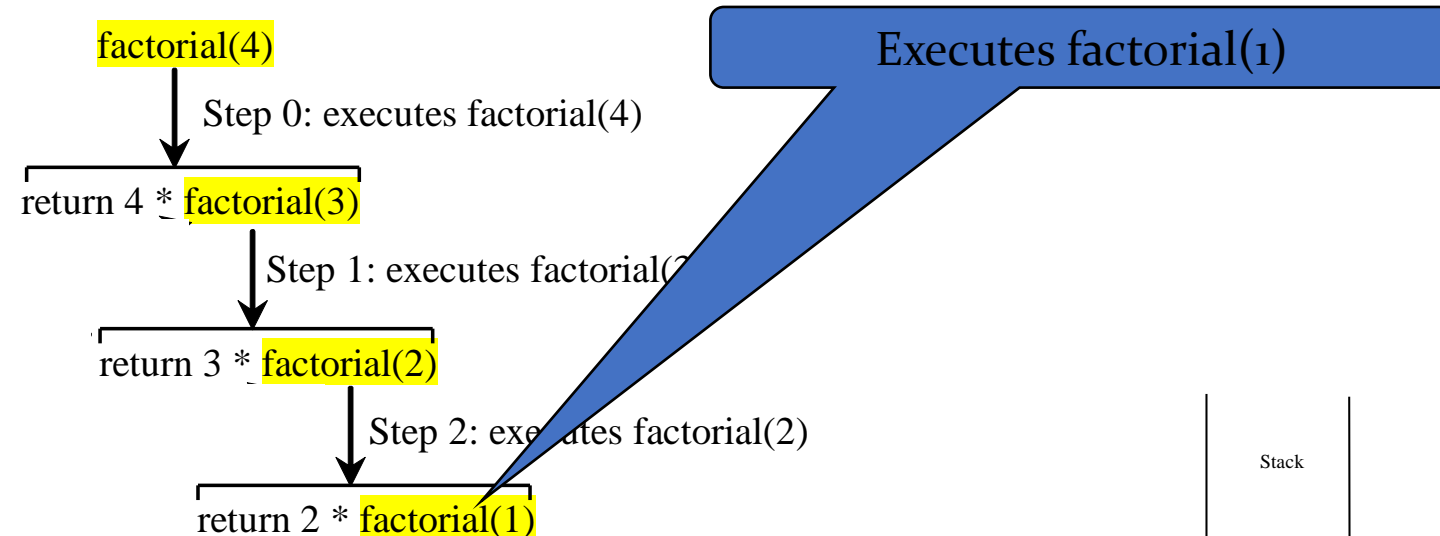


Recursive Factorial



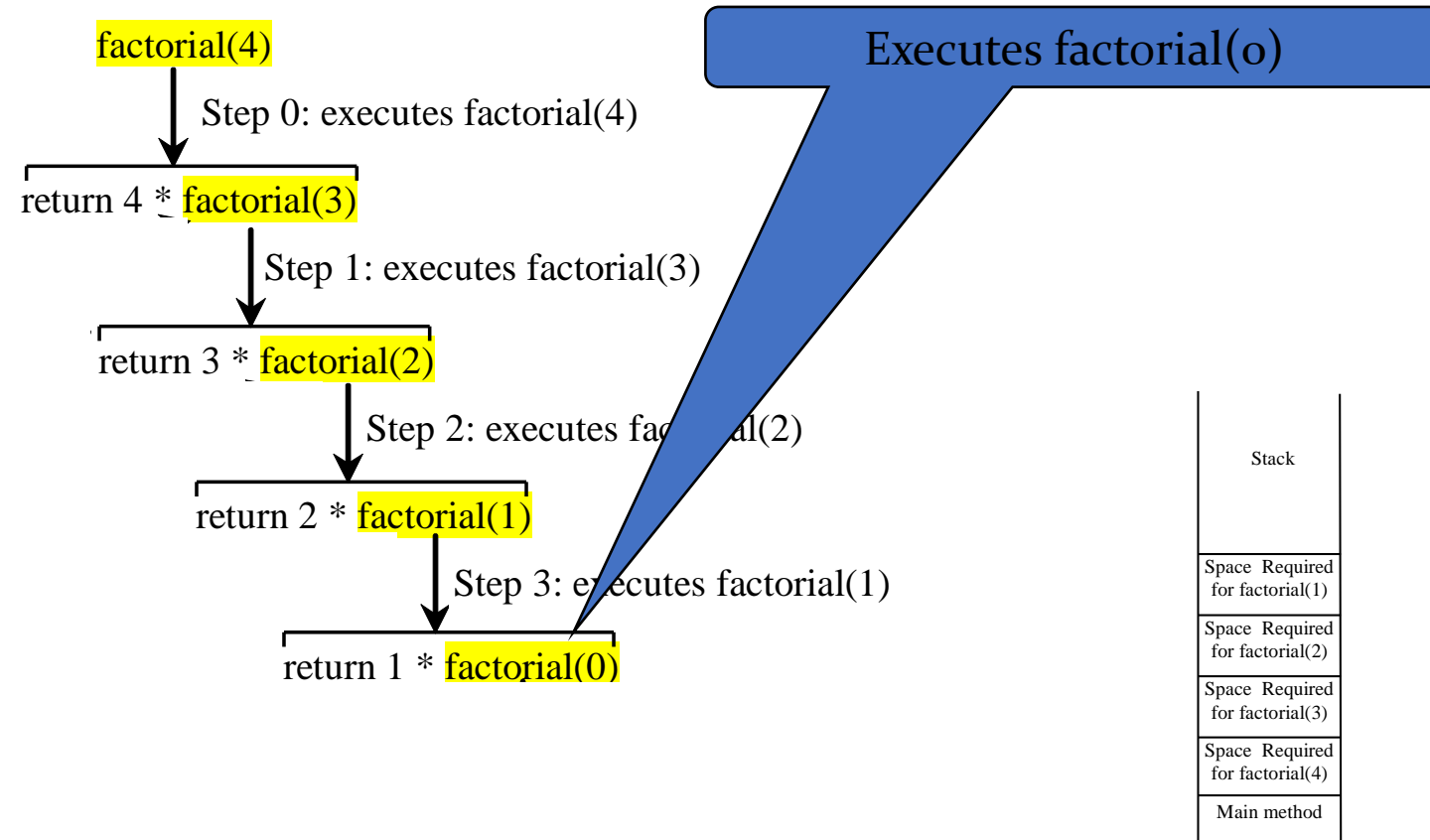
Stack
Space Required for factorial(3)
Space Required for factorial(4)
Main method

Recursive Factorial

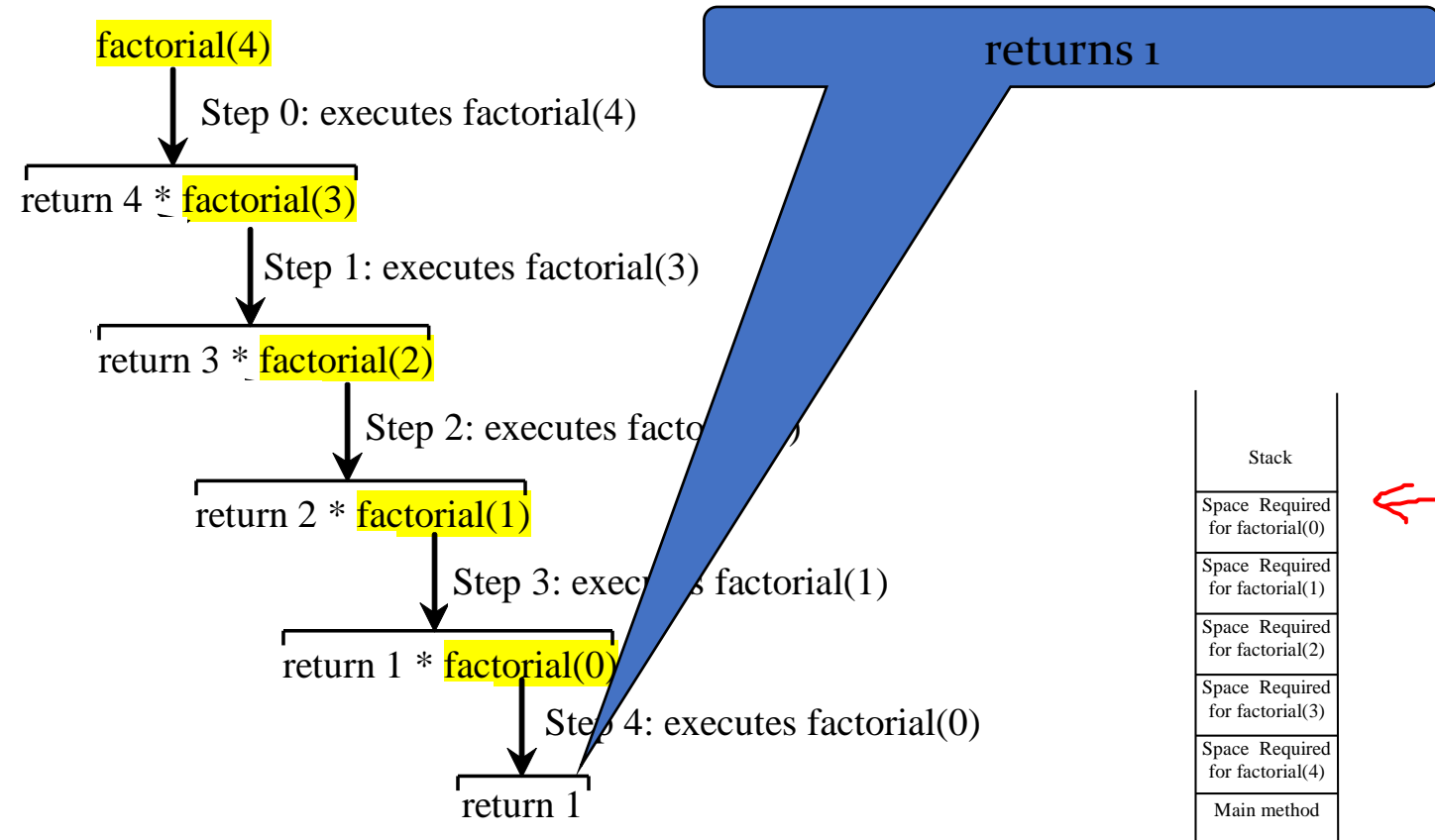


Stack
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

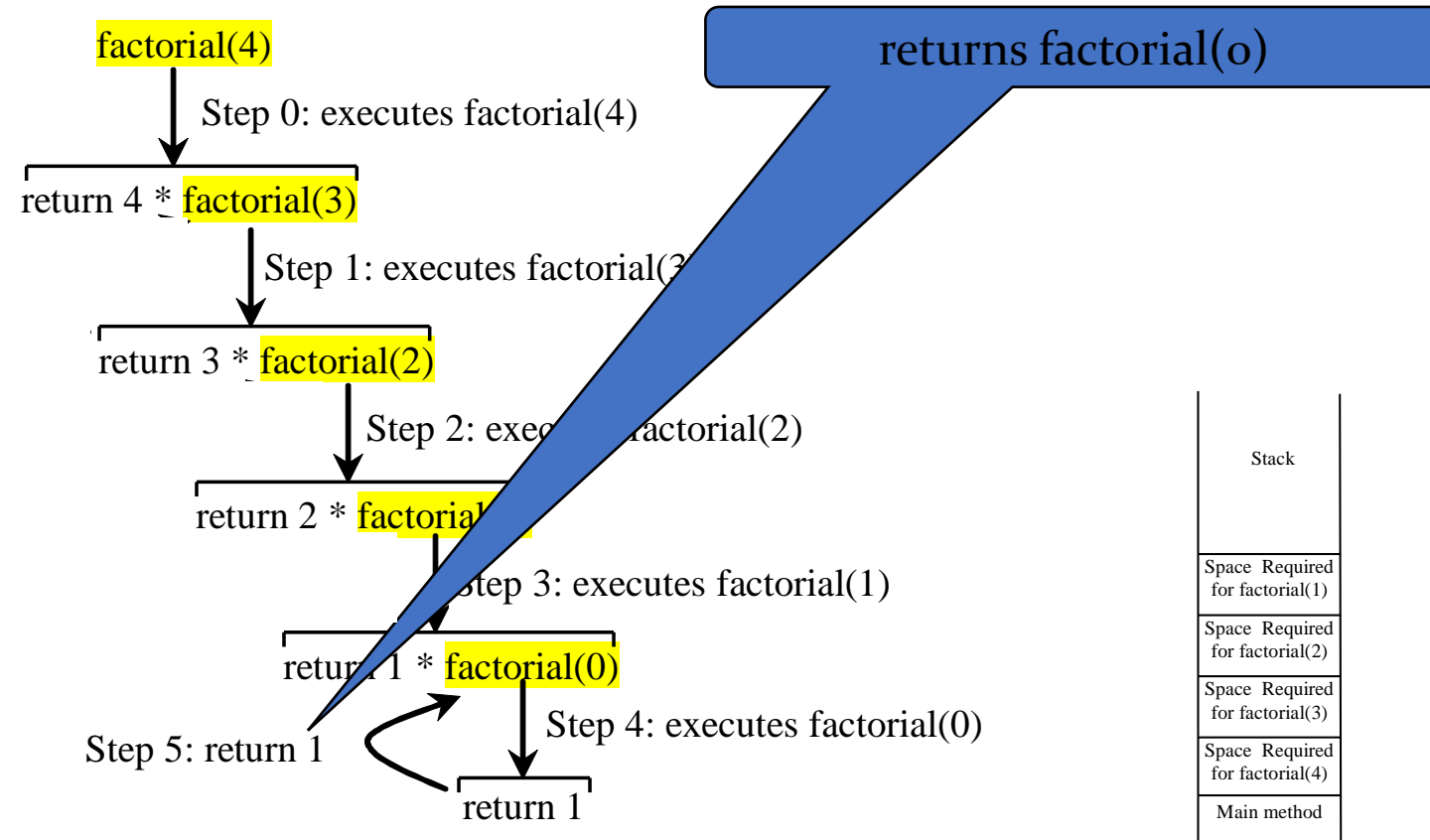
Recursive Factorial



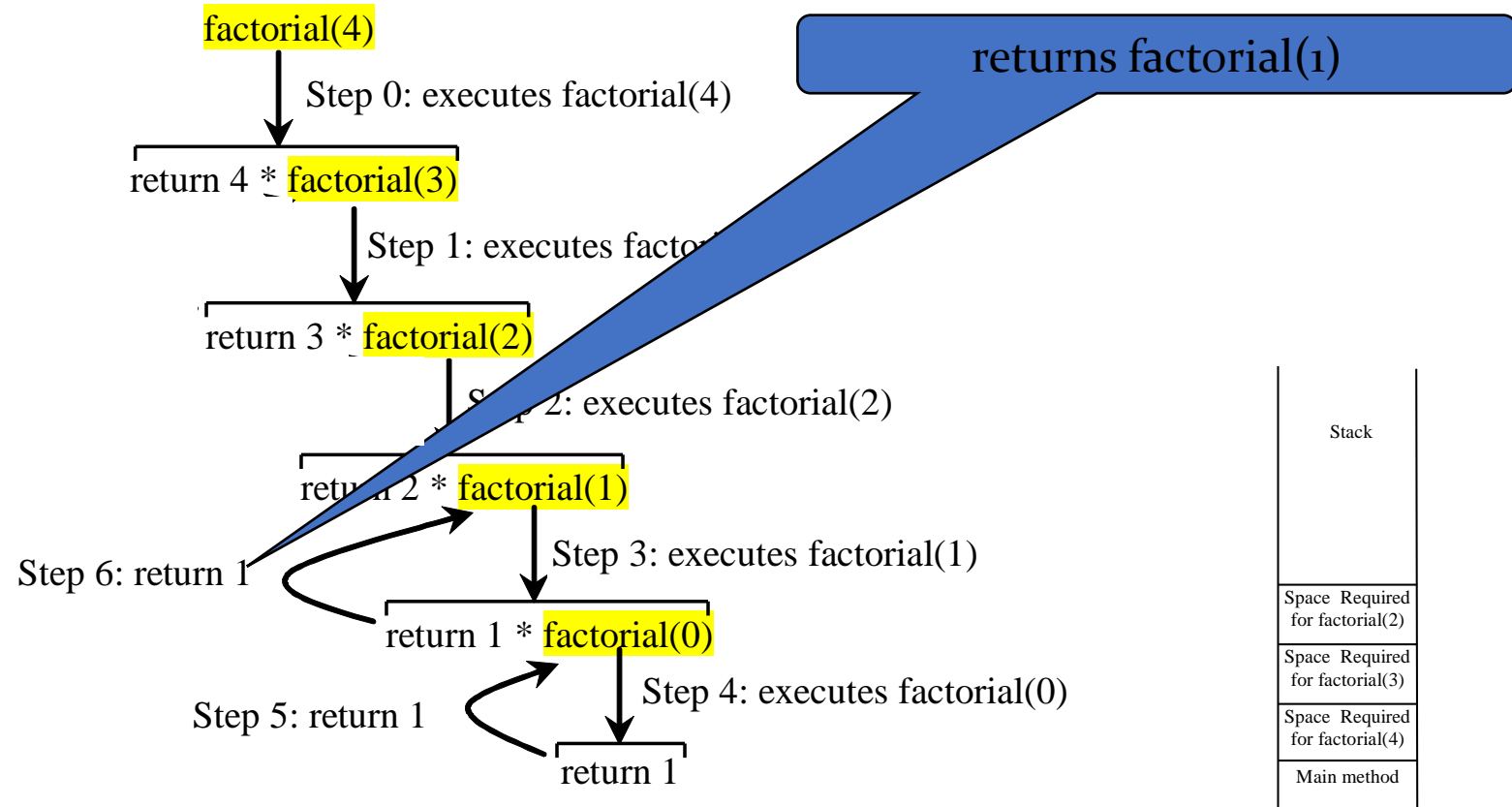
Recursive Factorial



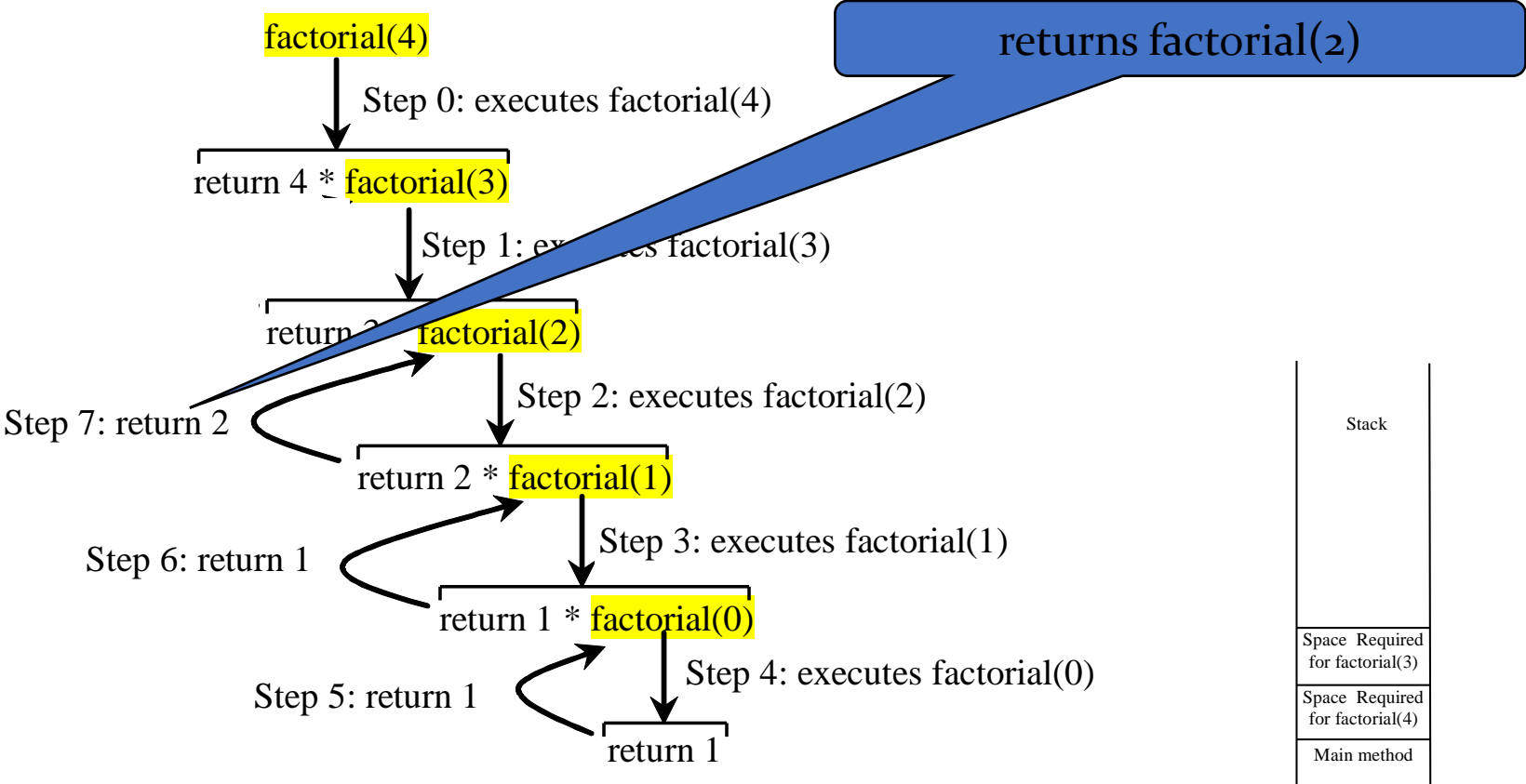
Recursive Factorial



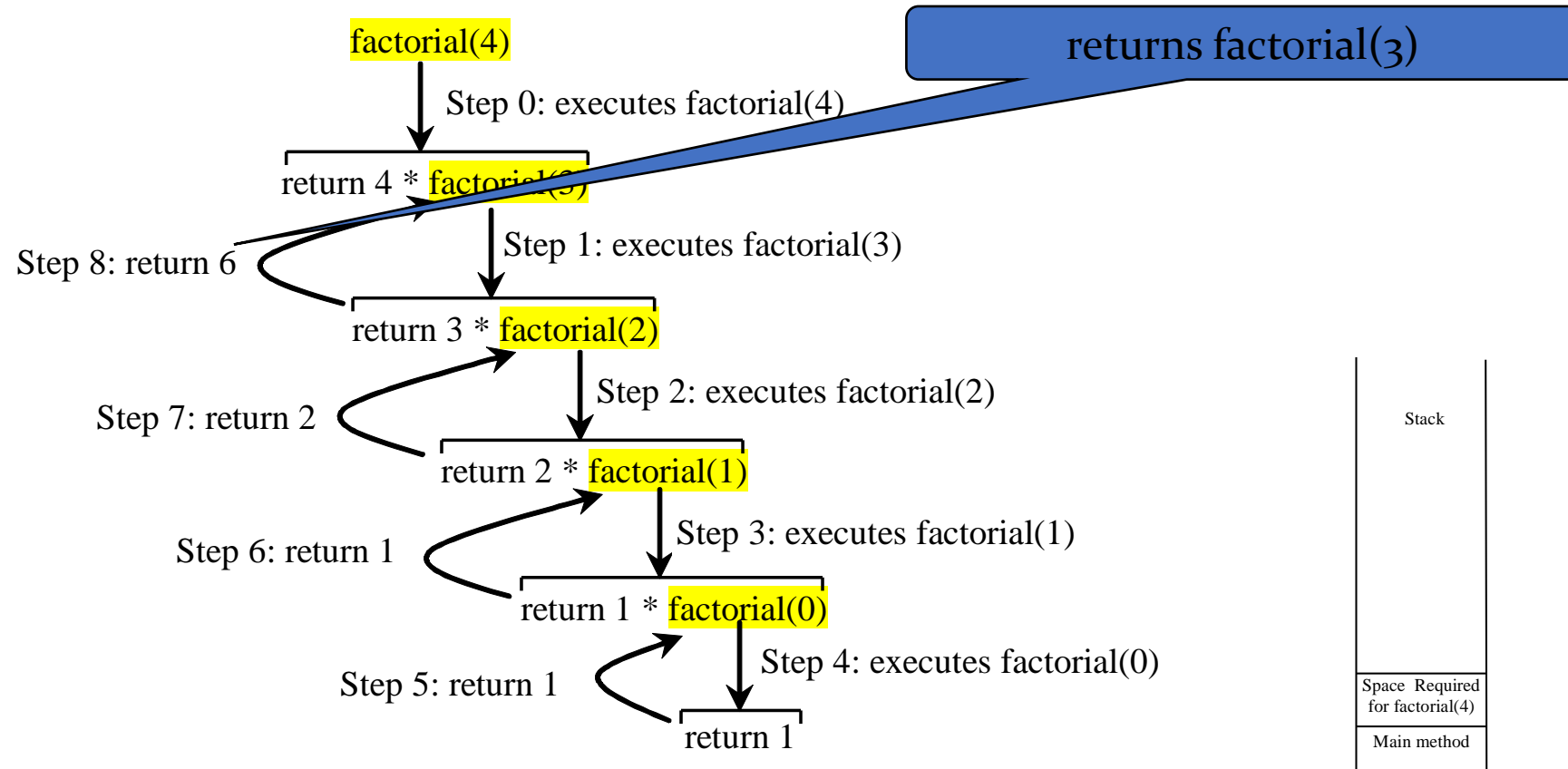
Recursive Factorial



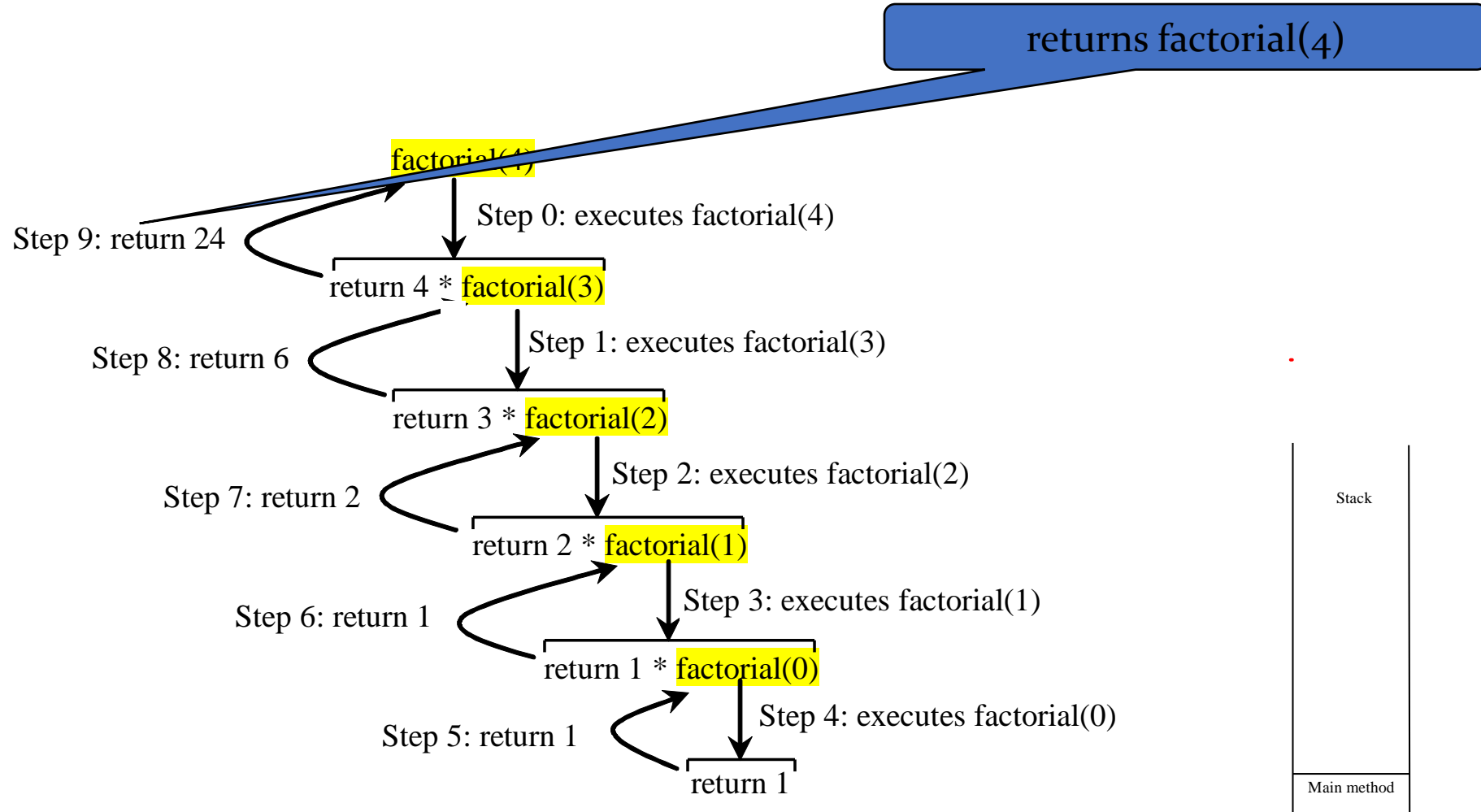
Recursive Factorial



Recursive Factorial



Recursive Factorial



Exercise

- Write your own recursive *int pwr()* function that takes two integers as arguments and returns the integer result.

- Prototype: `int pwr(int base, int exp);`

`pwr(2, 10)`

$$2^{10} = 1024$$

Demo...

Pros and Cons of Recursion

- Pros
 - Readable
 - Sometimes easier to conceptualize for problems that have many moving parts
- Cons
 - Efficiency
 - Memory usage
 - Each call to the function makes a new function stack frame (see previous slides)

Today's topic(s)

- Templates
- Standard Template Library (STL)
- Linked List

Templates

- How would you write a function to swap two ints?

```
void swap (int& a, int& b){  
    int temp = a;  
    a = b;  
    b = temp;  
}
```

- What if we also want to swap two floats?

```
void swap (float& a, float& b){  
    float temp = a;  
    a = b;  
    b = temp;  
}
```

- Two doubles? Two chars? Two strings? Two Animal objects?...

Function Templates

- Useful when have a general algorithm which doesn't change even if types change
- **Algorithm Abstraction:** expressing algorithms in a very general way so that we can ignore incidental detail and concentrate on the substantive part of the algorithm
- Classic example: swap
 - We can create a template function which can take any type

```
template <class T>  
void swap (T& a, T& b) {  
    T temp = a;  
    a = b;  
    b = temp;  
}
```

Function Templates

- `template <class T>`
 - Referred to as template prefix
 - Tells the compiler that the definition that follows is a template
 - T is a type parameter
- To call this function template, we can explicitly specify our template parameter using angle brackets:
 - `swap<int>(i, j); // where i and j are ints`
 - `swap<float>(x, y); // where x and y are floats`
 - `swap<Animal>(a1, a2); //where a1 and a2 are Animals`
- Since `swap()` takes parameters of the template type T, we don't need to explicitly specify the template type, i.e. these also work:
 - `swap(i, j); // where i and j are ints`
 - `swap(x, y); // where x and y are floats`
 - `swap(a1, a2); //where a1 and a2 are Animals`

Function Templates

- We can write function templates that include any number of template parameters, e.g:

```
template <class T, class U>
void print_two_things(T first, U second) {
    cout << first << second << endl;
}
```

And we can call it as before:

```
print_two_things<string, int>("number: ", 1);
print_two_things(2.5, 'e');
```


Note:

- The compiler generates a new implementation of the template for each type with which it is used.
 - This means concrete implementations of templates (i.e. int, float) are not created until compile time
- Therefore, we cannot explicitly compile template implementations into object files from .cpp files.
 - In fact, we can't separate template implementations into separate .cpp files at all
 - Instead, we need to write template implementations either in the same file in which they are used or else in a header (.h) file

Template Classes

- Work the same way as templated functions
- All functions within the class will operate on the provided types
- Scope with `ClassName<T>::functionname()`
- Each function needs the Template prefix

Today's topic(s)

- Template
- Standard Template Library (STL)

Standard Template Library (STL)

- C++ STL can be broken down into:
 - **Containers** – general purpose data structures (templates) for holding things
 - **Iterators** – special classes for traversing containers
 - **Algorithms** – sorting, searching, etc.
- Iterators make it possible to run the algorithm on the containers
- The STL is a great resource:
 - It contains a wide variety of very useful structures and algorithms
 - It is well-implemented, which means the structures and algorithms perform very efficiently
 - In general, it allows us to avoid re-inventing the wheel

Introducing STL Containers

- Predefined templates that can store any type of data
- The appropriate container will be dictated by the application requirements
- Example considerations:
 - Does the data need to be stored?
 - How will the data be accessed?
 - Front to back
 - Randomly?
 - Will additional data ever need to be added or removed?
- Careful planning will allow you to write clean, efficient code

Types of Containers

- Sequential containers (vector, deque, list)
 - Programmer controls the order of the elements
- Associative containers (map, set, multimap, multiset)
 - Position of elements is controlled by container
 - Elements are generally accessed by using a “key”
- Adapters (stack, queue)
 - Use an existing type of container to build other types
 - In this context, we call these “Abstract Data Types”

Examples of C++ Containers

- `<array>` - stores a constant amount of data in contiguous memory
- `<vector>` - An array that can be resized
- `<list>` - Linked list that stores data in non-contiguous memory
- `<set>` - An ordered collection of items
- `<queue>` - Stores data & returns it in the order it was received
 - First in, first out
- `<stack>` - Stores data & returns it in the opposite order that it was received
 - First in, last out
- Generally, it is a good idea to refer to the STL [documentation](#) before starting a project