CS 261-020
Data Structures

Lecture 5

Complexity Analysis

1/30/24, Tuesday

Oregon State University

Odds and Ends

- Assignment 1 past due
  - 70% if submit by tonight

- Recitation 4 posted
- Assignment 2 will be posted after the 70% due of asm1

- Midterm time update:
  - ~~Previous: Tue of week 5 (Feb 6)~~
  - Now: Tue of week 6 (Feb 13) during lecture time

Example:

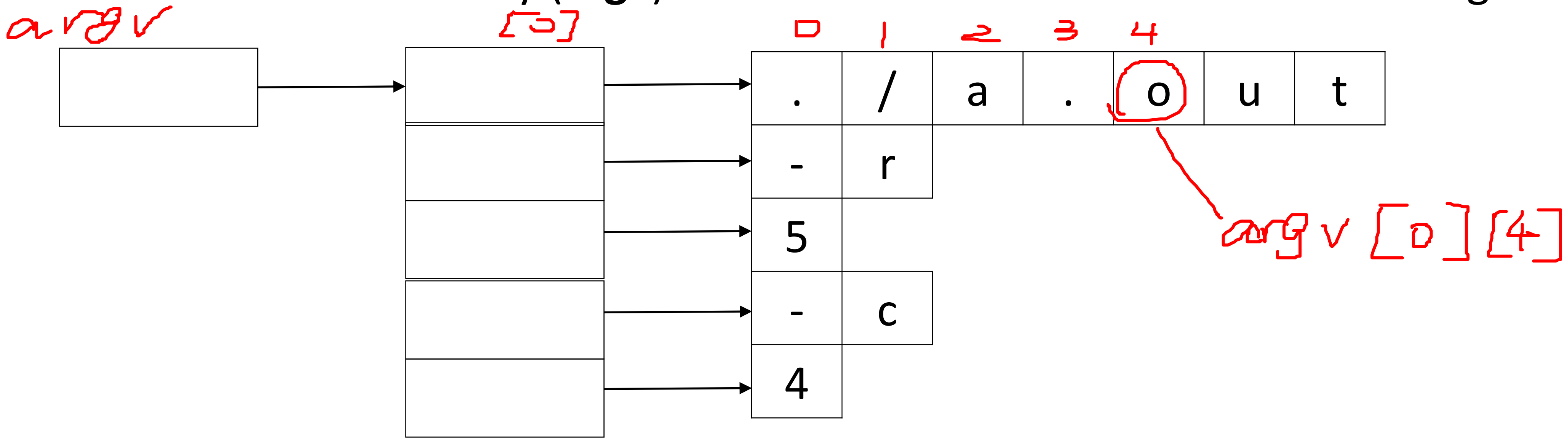
5

- What is the value of **argc** if user entered this command to run the program?

```
./a.out -r 5 -c 4
```

1 2 3 4 5

- What does the 2-d array (**argv**) look like for the above command-line arguments?



Lecture Topics:

- Complexity Analysis

How to compare Data Structures?

- We have different data structures, how to compare them?

- We want a way to characterize runtime or memory usage that is completely **platform-independent**
    - i.e. does not depend on hardware, operating system, programming language, etc.

Complexity Analysis

- Use Complexity Analysis to help make platform-independent comparisons of data structures
  - Also known as Big O

- To do this, we describe how a data structure's runtime or memory usage changes relative to a change in the input size (**n**)
  - Importantly, we want to describe how data structures behave in the limit, as n approaches ∞ (infinity)

$\lim_{n \to \infty}$

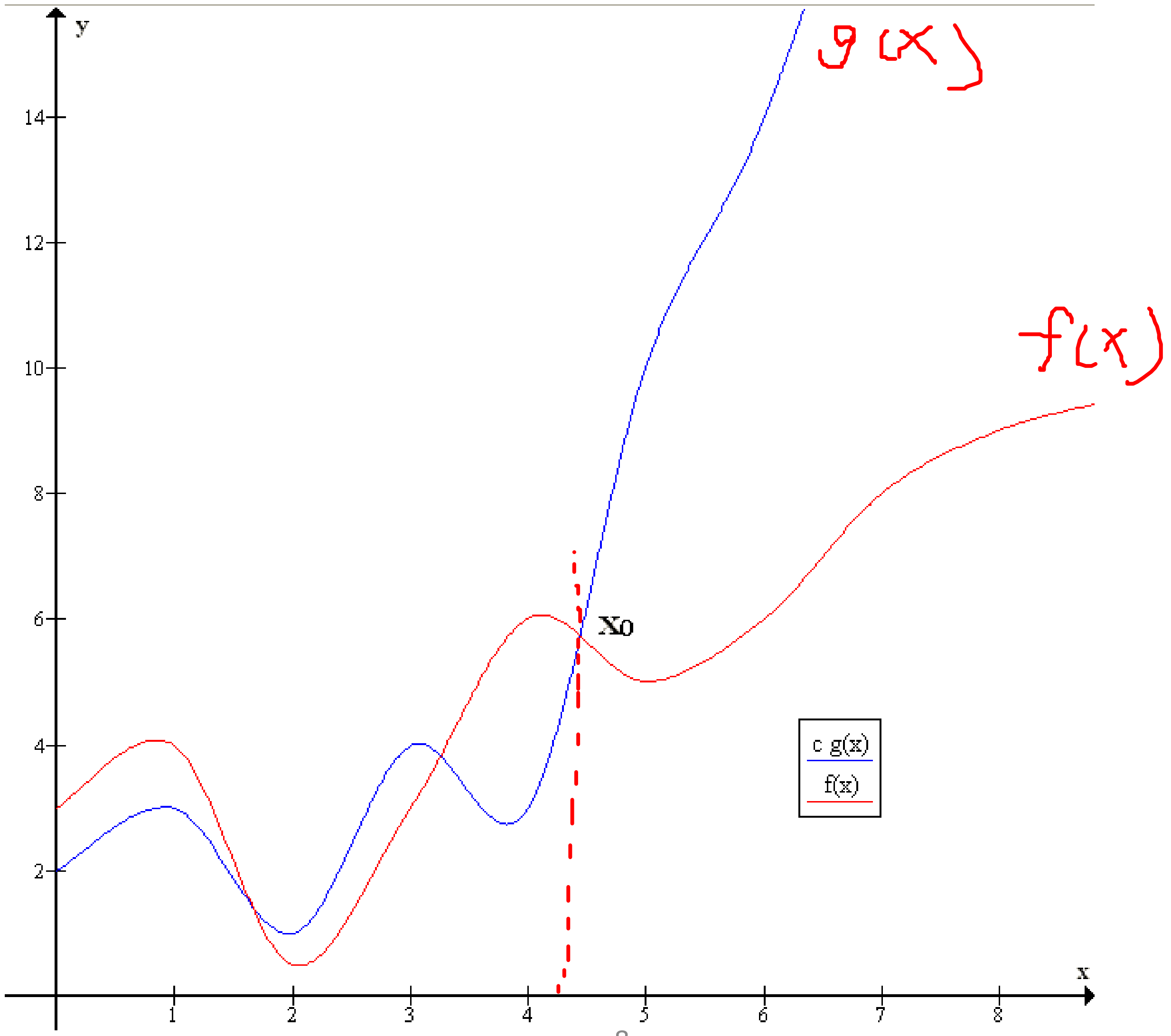
Big O

- We use Big O notation to assess a data structure or algorithm’s performance.

- Big O notation: a tool for characterizing a function in terms of its growth rate
  - Indicate an upper bound on the function’s growth rate, known as growth order

Big O

g(x) provides an upper bound on *f(x)*

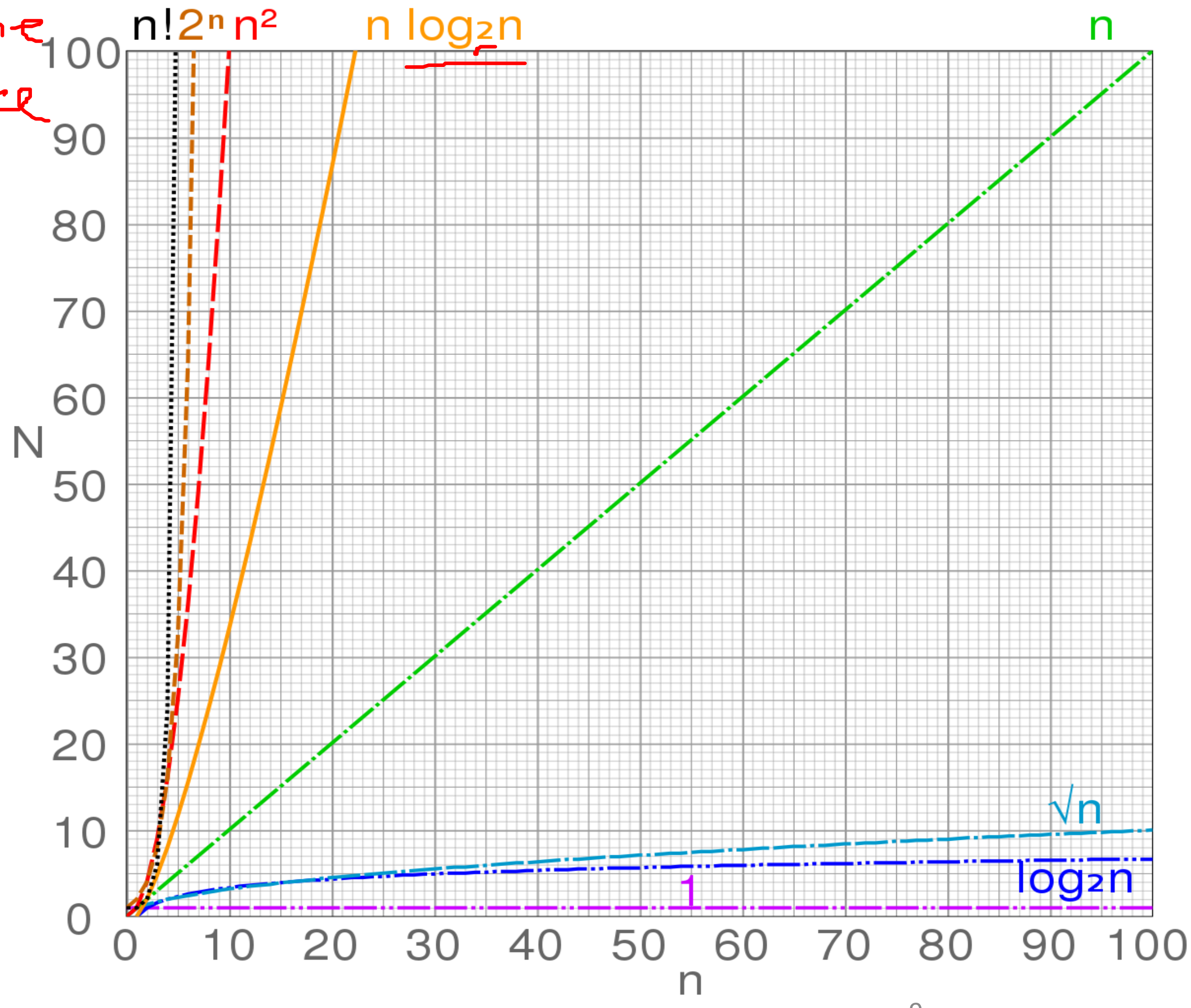


for x : x > x₀
g(x) > f(x)

g(x) is *O(f(x))*

Common growth order functions

time /space



input

Common growth order functions

log 2

- O(1) – constant complexity
- O(log n) – log-n complexity
- O(√n) – root-n complexity
- O(n) – linear complexity
- O(n log n) – n-log-n complexity
- O(n^2) – quadratic complexity
- O(n^3) – cubic complexity
- O(2^n) – exponential complexity
- O(n!) – factorial complexity

Compute Runtime Complexity

input size (n)

```
int sum = 0;
for (i = 0; i < n; i++) {
        sum += array[i];
}
return sum;
```

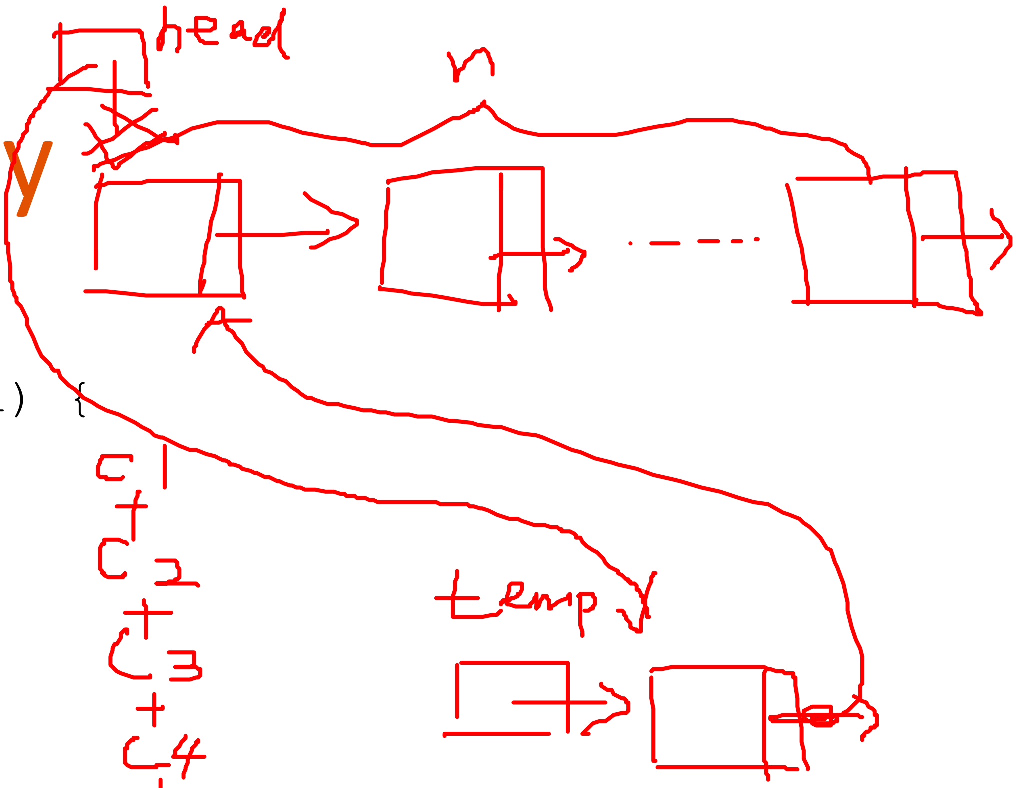
$c_1 + c_2 * n + c_3$

- The instruction `int sum = 0;` executes in some constant time c1 independent of n
- Each iteration of the loop executes in some constant time c2, and this happens n times
- The return statement executes in some constant time c3 independent of n
- So runtime is c1 + c2*n + c3
- c1, c2, and c3 depend on the particular computer running this function, so we ignore them to figure out run-time complexity
- Thus, this function grows on the order of n, a.k.a. its run-time complexity is **O(n)**

Compute Runtime Complexity

input size : n → # of nodes

```
struct node* push (struct node * head, int val) {
        struct node *temp = ~~new~~ malloc node;
        temp->val = val;
        temp->next = head;
        head = temp;
        return head;
}
```



c1 + c2 + c3 + c4 + c5

- Every instruction in this function executes in some constant time, independent of n
- Thus we ignore them to figure out runtime complexity.
- Complexity: O(c1+c2+c3+c4+c5) = **O(1)**

More examples

- Loops are one of the main determinants of a program's complexity

- ```
  for (int i = 0; i < n; i++) {
        ...
  }
  ```
  $O(1)$ $O(n)$

- ```
  for (int i = n; i > 0; i/=2) {
        ...
  }
  ```
  $2^i = n$ $i = \log_2 n$ $O(\log n)$

- ```
  for (int i = 0; i*i < n; i++) {
        ...
  }
  ```
  $O(\sqrt{n})$

  $i^2 < n$

  $i < \sqrt{n}$
  ```

More examples

n

- ```
  for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
                  ...
        }
  }
  ```

n

i: 0, 1, ..., n-1

inner loop: $n + n + n + \dots + n$

$\underbrace{n+n+n+ \cdots + n}_{n \text{ terms}} = n * n = n^2$

$O(n^2)$

$\log n$

- ```
  for (int i = n; i > 0; i/=2) {
        for (int j = 0; j < n; j++) {
                  ...
        }
  }
  ```

n

$O(n \log n)$

Determining a program's complexity

```
void bubble_sort(struct node *head, int size) {
        …
        for (int j = 1; j < size; j++) {
                for (int i = 0; i < size-j; i++){
                        if (current->val > current->next->val)
                                //swap
                        //move current to next node
                }
                current = head;
        }
}
```

n

O(1)

O(1)

O(1)

O(1)

j	# of iter
1	n − 1
2	n − 2
3	n − 3
⋮	⋮
n−3	n − (n−3) = 3
n−2	2
n−1	1

- Number of iterations:
- O((n-1) + (n-2) + (n-3) + … + 2 + 1)
- $= O(\frac{[(n-1)+1]*(n-1)}{2})$
- $= O\left(\frac{n^2-n}{2}\right)$
- $= O(n^2 - n)$ Is this the final answer?

$n(n-1)$

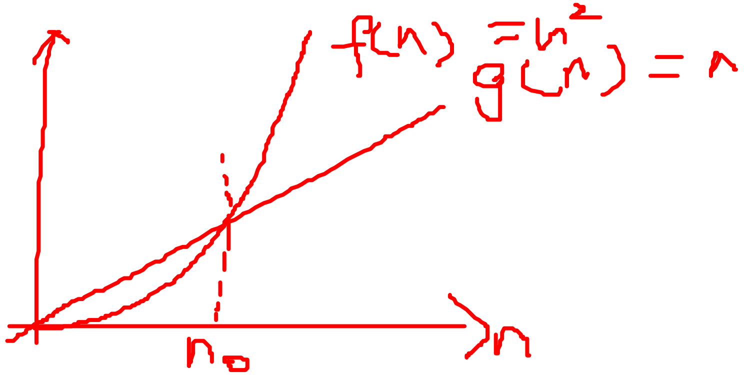
$\frac{[(n-1)+1](n-1)}{2}$

$O(n^2)$

$\lim (n^2 - n)$

$n \to \infty$

Dominant components



- When a growth order function has additive terms, one of those will dominate the others
  - Specifically, function *f(n)* dominates *g(n)* if n0:n>n0, *f(n)* > *g(n)*

- In these cases, we simply ignore the non-dominant terms
  - i.e. $n^2 - n$, n^2 dominates n, so we ignore n, and we say this complexity is $O(n^2)$

Dominant components

- Example: If an algorithm grows on the order of $n^2 + n + log\ n + 1$, what is the complexity of the algorithm using big O notation?

$O(n^2)$

- Takeaway: When loops are executed in sequence, the loop with the highest runtime complexity will determine the overall runtime complexity of the whole function.

Dominant components

- Example: What's the runtime complexity of the following?

```
for (i = 0; i < n; i++) {
   ...
}


for (i = 0; i < n; i++) {
        for (j = 0; j < n; j++) {
                ...
        }
}


for (i = 0; i < n; i++) {
   ...
}
```

$$n + n^2 + n \Rightarrow O(n^2)$$

Calculating time from Big O

$f(n) = n$

t_1 n_1 t_2

- If a O(n) algorithm takes 32ms to sum 10,000 elements, how long will it take to sum 20,000? n_2

$$\boxed{\frac{f(n_1)}{f(n_2)} = \frac{t_1}{t_2}} \Rightarrow \frac{n_1}{n_2} = \frac{t_1}{t_2} \Rightarrow \frac{10000}{20000} = \frac{32}{t_2} \Rightarrow \frac{1}{2} = \frac{32}{t_2}$$

$t_2 = 64ms$

  - For an *O(n)* algorithm, if size doubles, execution time doubles.

- What if this algorithm has O(n^2) complexity? $f(n) = n^2$ $n_2 = 2 \times n_1$

$$\frac{f(n_1)}{f(n_2)} = \frac{t_1}{t_2} \Rightarrow \frac{n_1^2}{n_2^2} = \frac{t_1}{t_2} \Rightarrow \frac{n_1^2}{(2n_1)^2} = \frac{t_1}{t_2} \Rightarrow \frac{\cancel{n_1^2}}{4 \cdot \cancel{n_1^2}} = \frac{t_1}{t_2}$$

$t_2 = 4 \times t_1 = 4 \times 32 = 128ms$

$f(n) = n^3$ $t_2 = 8 \times t_1$

  - Runtime goes up by a factor of 4. $8 = 2^3$

Calculating time from Big O

- Ex. Merge sort, which is an *O(n log n)* algorithm, takes 96ms to sort an array of size 4000. Given this result, approximately how long merge sort will take to sort an array of size 1,000,000? In seconds (S)
- Hint: $4000 \approx 2^{12}$, $1{,}000{,}000 \approx 2^{20}$

Worst case, Best case, and avg. case

- Note that the worst case, best case, and average case complexities of a data structure or an algorithm can differ, for example:

```
int linear_search(int q, int* array, int n) {
        for (int i = 0; i < n; i++) {
                if (array[i] == q) {
                        return i;
                }
        }
        return -1;
}
```

  - Worst case: O(n): if q appears to be the last element / does not exist
  - Best case: O(1): if q appears to be the first element
  - Avg. case: O(n): run about n/2 iterations, drop ½

Real-world Consideration

- Your program will only perform as well as your design
  - Constant factors can still play a part
- Suppose you have two data structure or algorithms perform the same task:
  - A) 1,000,000n → O(n)
  - B) 2 n^2 → O(n^2)
  - Which one is better?
    - It depends

Complexity of dynamic array insertion

- Recall: dynamic array insertion
  - Case 1: if size < capacity
    - Insert the new element
  - Case 2: if size == capacity
    - Step 1: allocate a new array that has twice the capacity
    - Step 2: copy all elements from data to new array
    - Step 3: delete the old data array and update data pointer
    - Step 4: Insert the new element

- Group Activity: What is the best-case, worst-case, and average case runtime complexities? Using Big O