CS 261-020
Data Structures

Lecture 6

Complexity Analysis

Stack, Queue, Deque

2/1/24, Thursday

Oregon State University

Odds and Ends

- Assignment 2 posted, due 2/11

- Due: Sunday 2/4 midnight
  - Quiz 2 (unlock after today's lecture)

Lecture Topics:

- Complexity Analysis
  - Array Insertion
  - List insertion & removal

- Stacks, Queues, and Deques
  - Linear ADTs

$\log_x x^y = y$

Calculating time from Big O

$f(n) = n \log n$

- Ex. Merge sort, which is an *O(n log n)* algorithm, takes 96ms to sort an array of size 4000 (n_1). Given this result, approximately how long merge sort will take to sort an array of size 1,000,000 (n_2)? (t_1, t_2) In seconds (S)
- Hint: 4000 ≈ 2^{12}, 1,000,000 ≈ 2^{20}

$$\frac{f(n_1)}{f(n_2)} = \frac{t_1}{t_2} \Rightarrow \frac{n_1 \log n_1}{n_2 \log n_2} = \frac{t_1}{t_2}$$

$$\frac{2^{12} \log_2 2^{12}}{2^{20} \log_2 2^{20}} = \frac{96}{t_2}$$

$$\frac{3}{2^8 \cdot 5} = \frac{96}{t_2}$$

$t_2 = 40960$ ms ≈ 41 S

Worst case, Best case, and avg. case

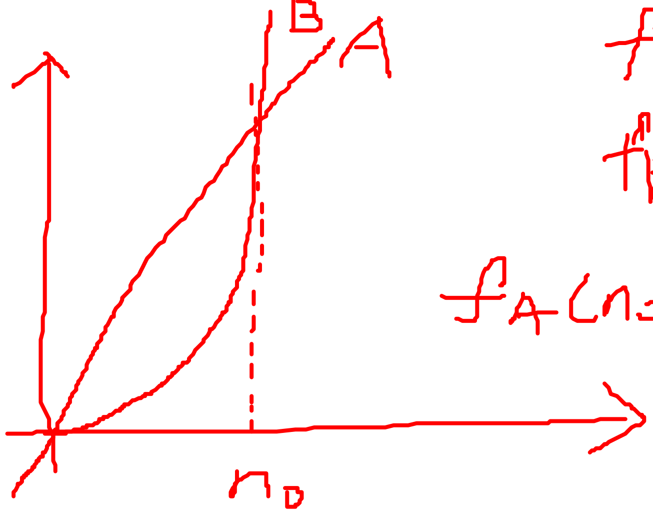
- Note that the worst case, best case, and average case complexities of a data structure or an algorithm can differ, for example:

```
int linear_search(int q, int* array, int n) {
        for (int i = 0; i < n; i++) {
                if (array[i] == q) {
                        return i;
                }
        }
        return -1;
}
```

  - Worst case: O(n): if q appears to be the last element / does not exist
  - Best case: O(1): if q appears to be the first element
  - Avg. case: O(n): run about n/2 iterations, drop ½

Real-world Consideration

- Your program will only perform as well as your design
  - Constant factors can still play a part
- Suppose you have two data structure or algorithms perform the same task:
  - A) 1,000,000n → O(n)
  - B) 2 n^2 → O(n^2)
  - Which one is better?
    - It depends



$f_A(n) = 1000000\,n$

$f_B(n) = 2n^2$

$f_A(n_0) = f_B(n_0)$

$n_0 = 500,000$

Complexity of dynamic array insertion

- Recall: dynamic array insertion
  - Case 1: if size < capacity
    - Insert the new element
  - Case 2: if size == capacity
    - Step 1: allocate a new array that has twice the capacity
    - Step 2: copy all elements from data to new array
    - Step 3: delete the old data array and update data pointer
    - Step 4: Insert the new element

- Group Activity: What is the best-case, worst-case, and average case runtime complexities? Using Big O

Complexity of dynamic array insertion

- Group Activity: What is the best-case, worst-case, and average case runtime complexities?

- Best case: when size < capacity
  - Write the new value into the next open space
  - Time it takes to run this operation doesn't depend on the size of the array (*n*)
  - Thus, O(1)
- Worst case, when size >= capacity
  - Require allocating a new array
  - Iterate through the *n* elements in the old array and copying them into the new array
  - Thus, O(n)

Complexity of dynamic array insertion

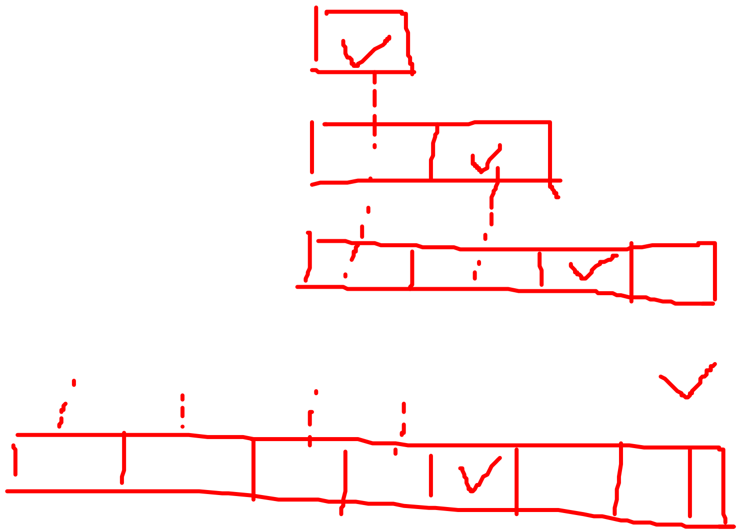
- Group Activity: What is the best-case, worst-case, and average case runtime complexities?

- How to determine average Case:
  - Use amortized analysis – a large cost is defrayed by spreading smaller payments over a period of time.
  - O(n) insertion cost (worst case) happens far less often than O(1) insertion cost (best case)
    - Since we double the capacity
  - Quantify the runtime complexity by aggregate analysis, by computing an upper bound **T** on the total cost of a sequence of **n** operations. Thus, average cost is **T / n**

Complexity of dynamic array insertion

- Assuming a dynamic array whose capacity starts at 1, doubled if resized. Perform a sequence of n insert. What's the total cost?

- 1st insertion: Write cost 1, copy cost 0
- 2nd insertion: Write cost 1, copy cost 1 (resize)
- 3rd insertion: Write cost 1, copy cost 2 (resize)
- 4th insertion: Write cost 1, copy cost 0
- 5th insertion: Write cost 1, copy cost 4 (resize)
- …..



Complexity of dynamic array insertion

- Assuming a dynamic array whose capacity starts at 1, doubled if resized. Perform a sequence of n insert. What's the total cost?

- Create a table:

Insertion # (resize # (*k*))	1	2 (1)	3 (2)	4	5 (3)	6	7	8	9 (4)	10	...
Write cost	1	1	1	1	1	1	1	1	1	1	...
Copy cost	0	1	2	0	4	0	0	0	8	0	...

$2^0 + 2^1 + 2^2 + 2^3 = 1111_2 = 10000_2 - 1 = 2^4 - 1$

Complexity of dynamic array insertion

Insertion # (resize # (*k*))	1	2 (1)	3 (2)	4	5 (3)	6	7	8	9 (4)	10	...
Write cost	1	1	1	1	1	1	1	1	1	1	...
Copy cost	0	1	2	0	4	0	0	0	8	0	...

- Total Write cost = n $1 \times n$
- Total copy cost:

$= 2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^{\log n - 1}$

$= 2^{\log n} - 1$

$= n - 1$

$1 + 2 + 4 + 8 + 16 + \cdots + 2^{\log n - 1}$

$1 + 2 = 3 = 4 - 1 = 2^2 - 1$

$1 + 2 + 4 = 7 = 8 - 1 = 2^3 - 1$

$1 + 2 + 4 + 8 = 15 = 16 - 1 = 2^4 - 1$

$1 + 2 + \cdots + 2^x = 2^{x+1} - 1$

$10^{\log_{10} x} = x$ $2^{\log_2 n} = n$

Complexity of dynamic array insertion

Insertion # (resize # (*k*))	1	2 (1)	3 (2)	4	5 (3)	6	7	8	9 (4)	10	...
Write cost	1	1	1	1	1	1	1	1	1	1	...
Copy cost	0	1	2	0	4	0	0	0	8	0	...

- Total cost = Total Write cost + Total copy cost:

= n + (n – 1)

= 2n – 1

$\frac{2n-1}{n} = 2 - \frac{1}{n}$

- Thus, average is (2n-1)/n = O(1)

Complexity of dynamic array insertion

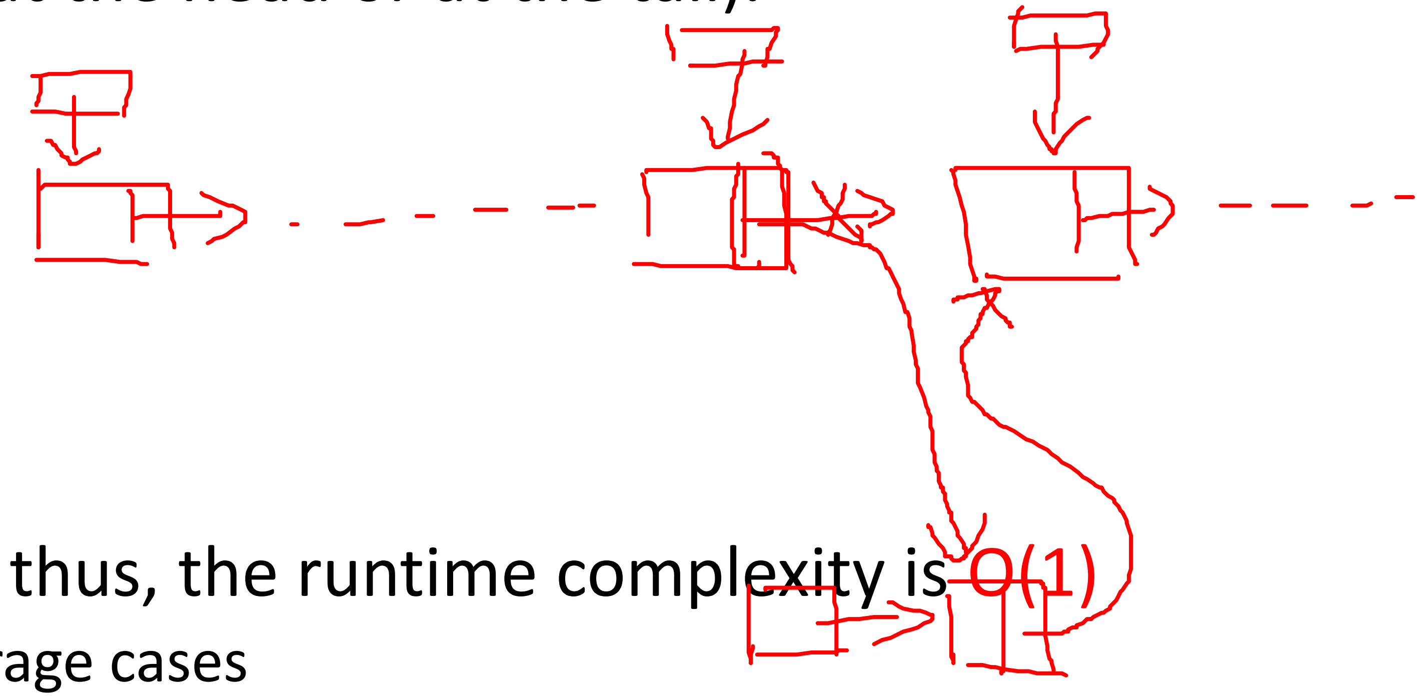
- Thus, average case is $(2n-1)/n = O(1)$

- On average, dynamic array insertion is a constant time operation.

Complexity of linked list insertion

- Assuming that we already know exactly where in the list we want to insert a new value (e.g. at the head or at the tail).

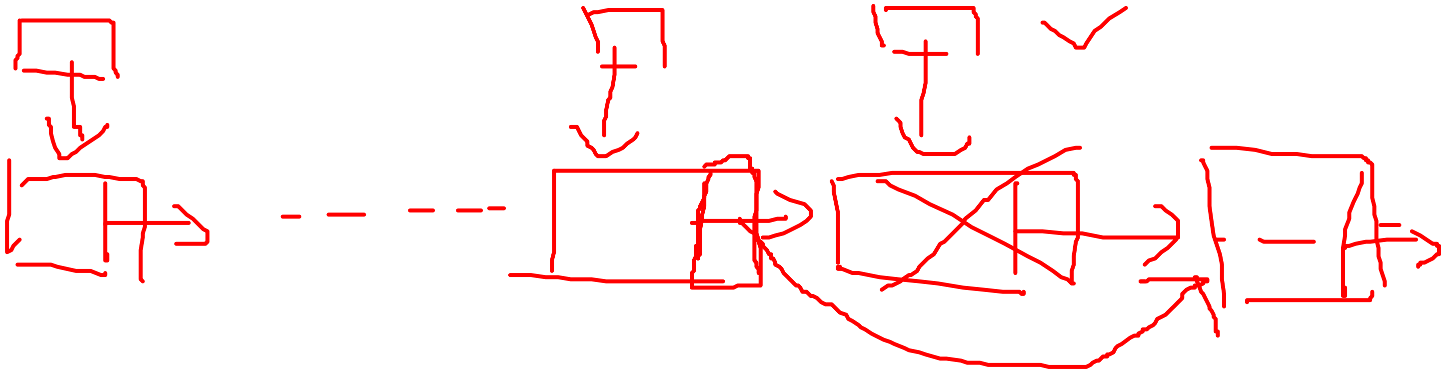
- Steps:
  - Allocating a new node
  - Updating pointers



- All run in constant time, thus, the runtime complexity is O(1)
  - For best, worst, and average cases

Complexity of linked list removal

- Assuming that we already know exactly where in the list we want to remove.



- Steps:
  - Updating pointers
  - Free the node
- All run in constant time, thus, the runtime complexity is O(1)
  - For best, worst, and average cases

Dynamic Array vs. Linked List

worst

	Dynamic Array	Linked List
Insertion	O(n)	O(1)
Removal	O(n)	O(1)
Access the nth element	O(1)	O(n)

Lecture Topics:

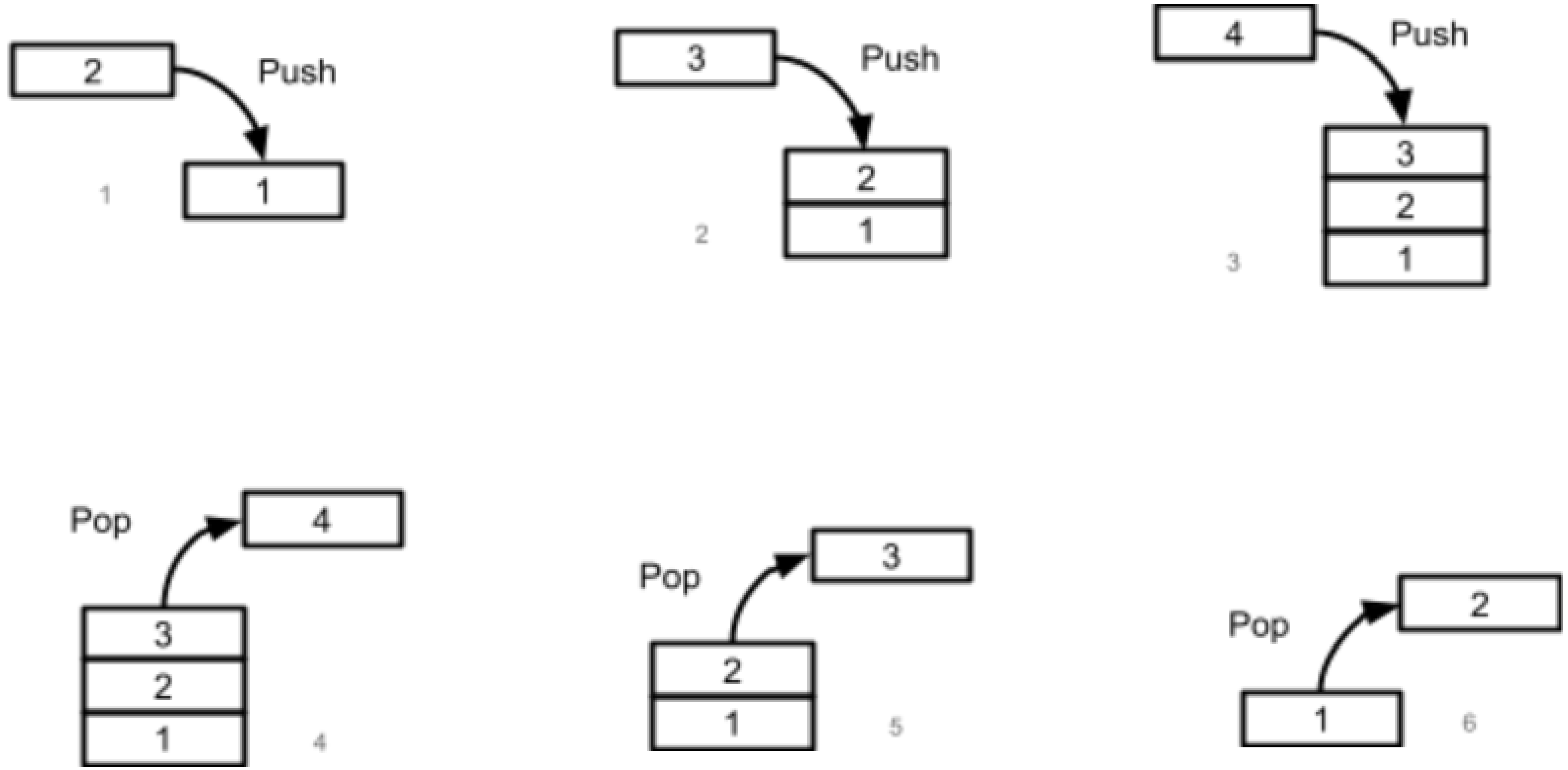
- Complexity Analysis
  - Array Insertion
  - List insertion

- Stacks, Queues, and Deques
  - Linear ADTs

Stacks

- A linear ADT that imposes a Last In, First Out (LIFO) order on elements
  - The last element inserted must be the first one to remove
  - Real life examples: a stack of books, a stack of dishes, web browser's "back" history, "undo" operation in a text editor
- A stack ADT has two ends: top and bottom
  - New elements can only be inserted at top
  - Only the element at the top may be removed
- Two main operations:
  - *Push* – inserts an element on the top
  - *Pop* – removes the top element

Stacks



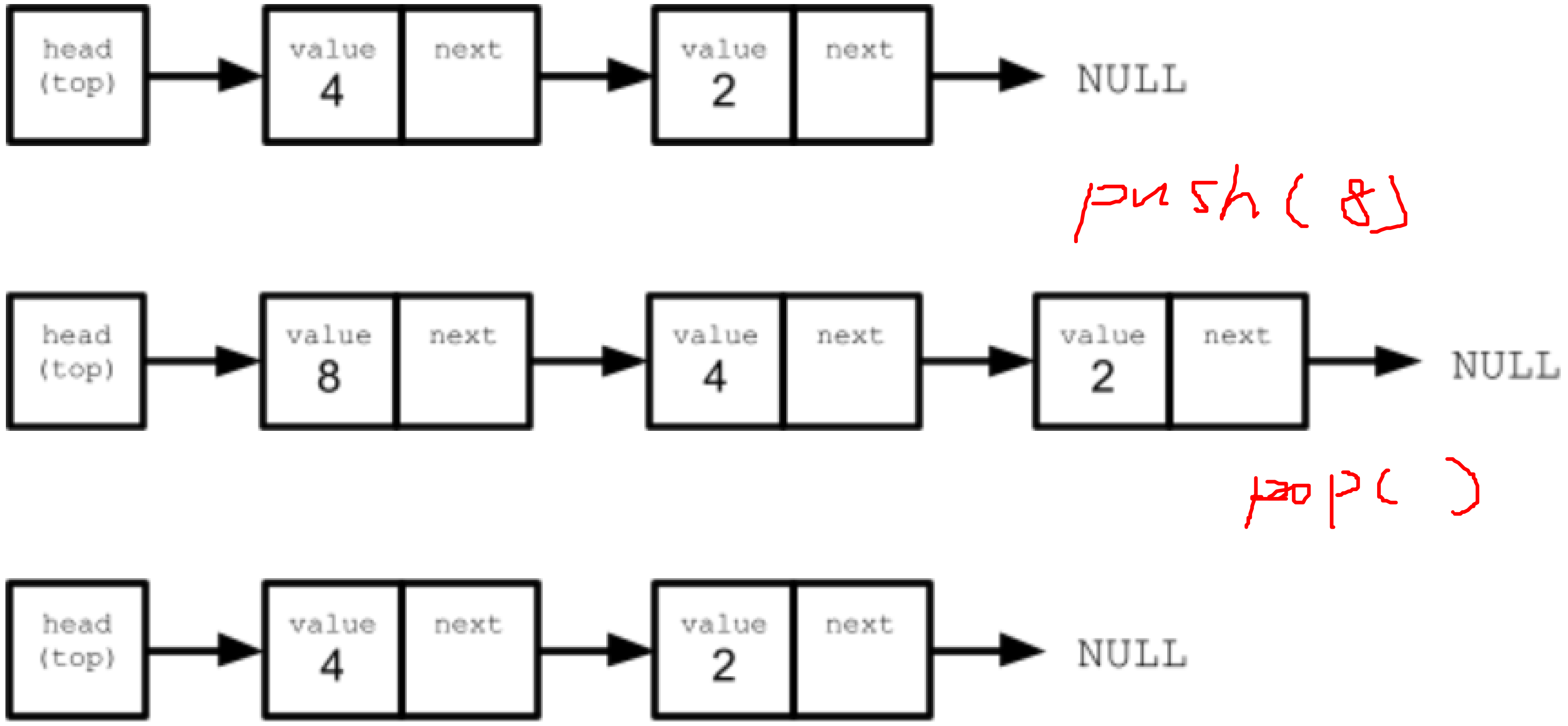
Implement Stack using Linked List

- Using a singly linked list, head of the list = the top of the stack

- When a value is pushed into a stack, it becomes the new head of the list

- When a value is popped, the current head of the list is removed
  - The next node becomes the new head

Implement Stack using Linked List



Implement Stack using Linked List

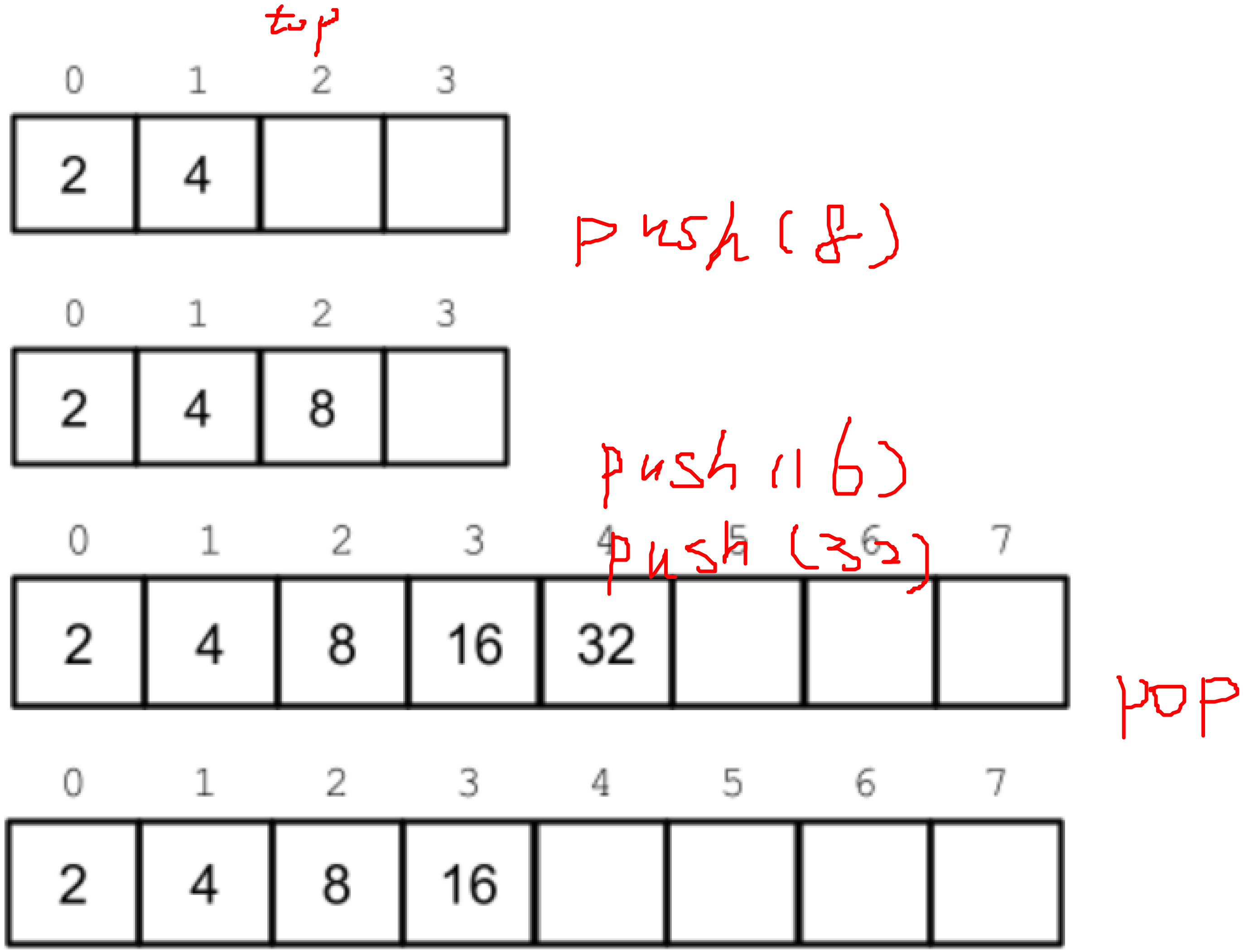
- Complexity Analysis:
  - Push() – O(1)
  - Pop() – O(1)

  *For all best-case, worst-case, and average-case

Implement Stack using Dynamic Array

- Using dynamic array, the end of the array = ~~head~~ top of the stack
- When a new element is pushed onto the stack, it is inserted at the end of the array
  - Resize if needed, as a normal dynamic array
- When an element is popped, the array's last element is removed

Implement Stack using Dynamic Array

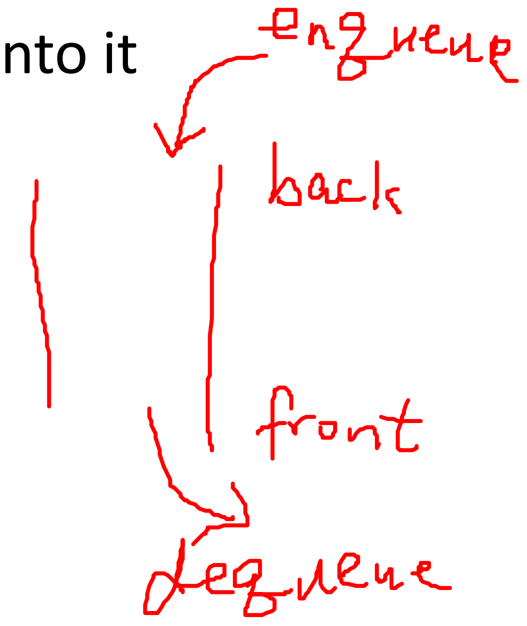


Implement Stack using Dynamic Array

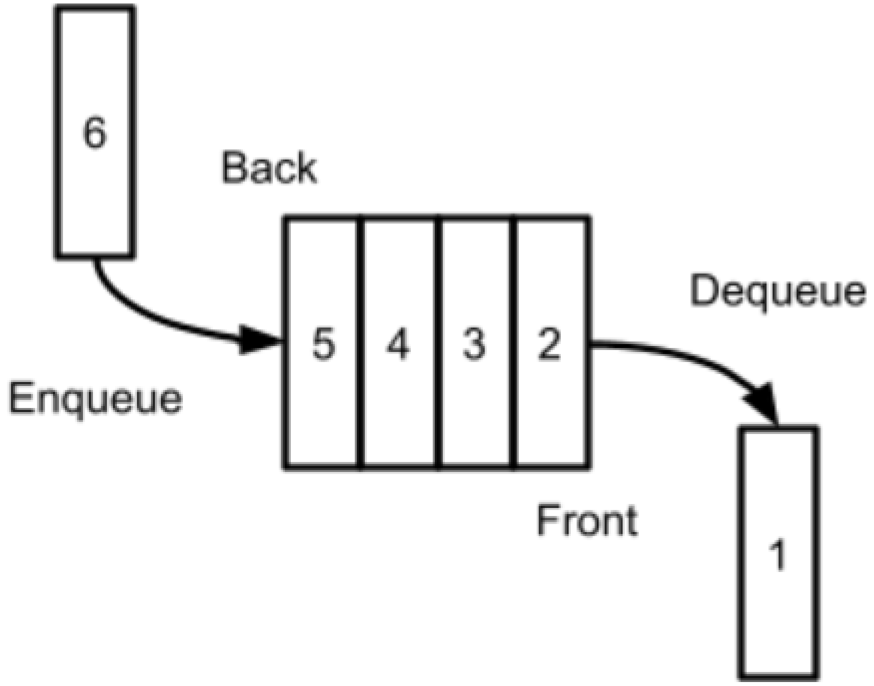
- Complexity Analysis
  - Pop() – O(1)
    - for all best-case, worst-case, and average case
  - Push()
    - O(1) Best-case and average case
    - O(n) worst-case (when resize is needed)

Queues

- A linear ADT that imposes a First In, First Out (FIFO) order on elements
  - The first element to be removed is the first one that was placed into it
  - Real life examples: a line of people waiting for check out
- A Queue ADT has two ends: front and back
  - Inserting elements to the back
  - Removing elements from the front
- Two main operations:
  - *Enqueue* – insert an element at the back
  - *Dequeue* – remove an element at the front



Queues



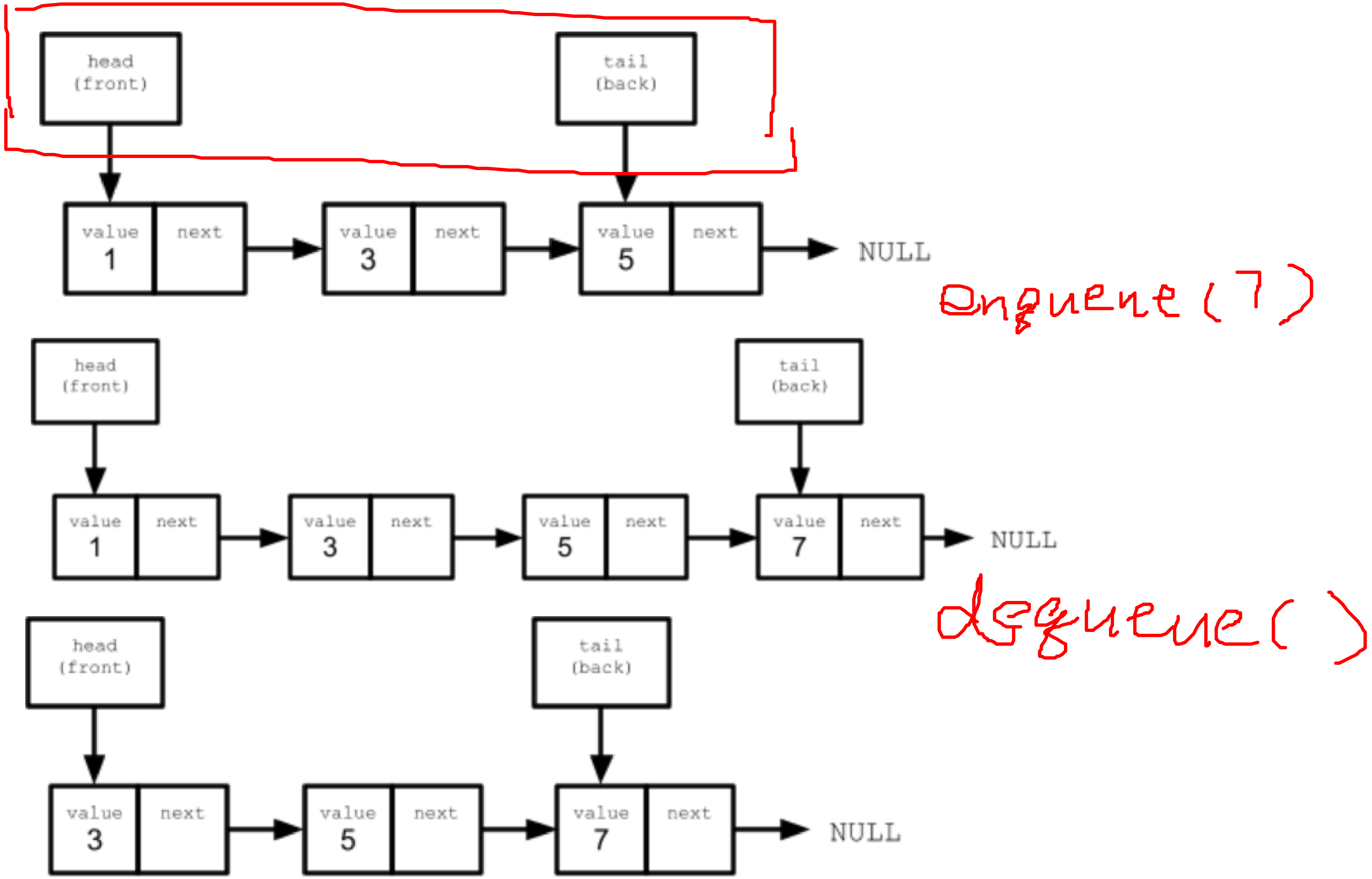
Implement Queue using Linked List

- Using a singly linked list. Must keep track of both the head and the tail of the list

- Enqueue onto the back → insert at the tail of the list

- Dequeue from the front → remove from the head of the list

Implement Queue using Linked List



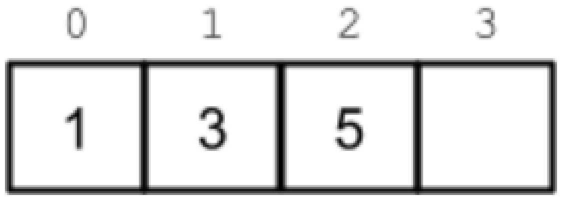
Implement Queue using Linked List

- Complexity Analysis:
  - enqueue() – O(1)
  - dequeue() – O(1)

  *for all best-case, worst-case, and average case

Implement Queue using Dynamic Array

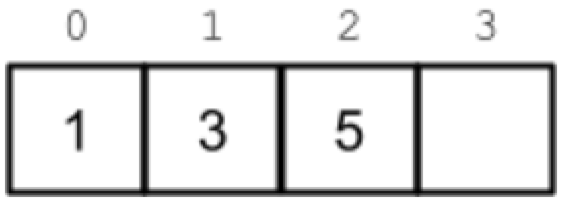
- Using a dynamic array,
  - Front of the queue = front of the array
  - Back of the queue = back of the array
- Ex. A queue with 3 values (1 at the front, 5 at the back)



- Enqueue a new value → insert it at the end of the array

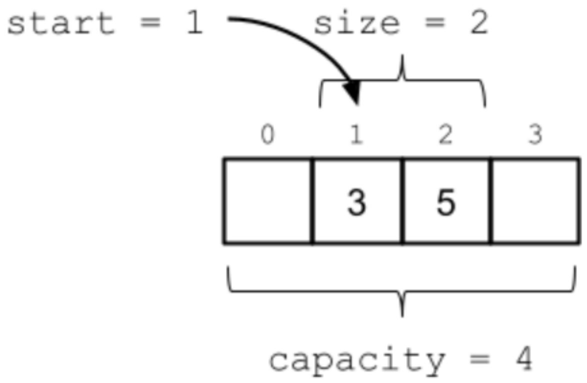
- What about dequeue?

Implement Queue using Dynamic Array

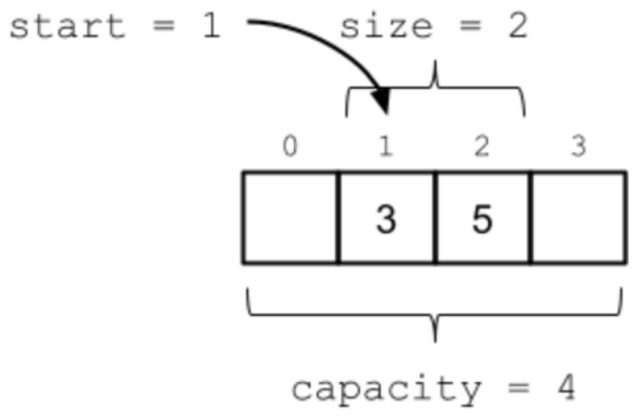


- Dequeue:
  - Option 1: remove the front, and shift all the remaining to left
    - Drawback: O(n) runtime complexity for each dequeue → NOT GOOD!!!

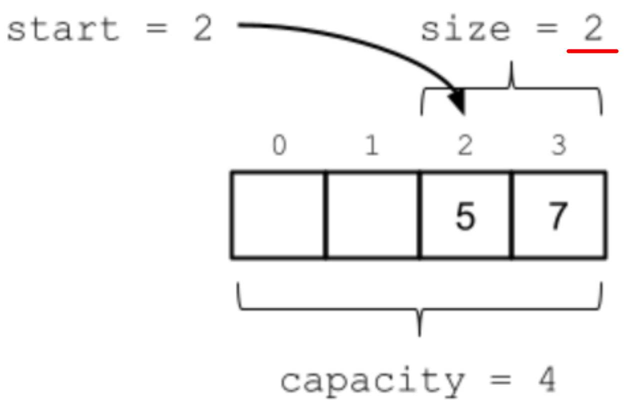
  - Option 2: allow the front of the queue to *"float" back* into the middle of the array.
    - Need to keep track of the start of the data



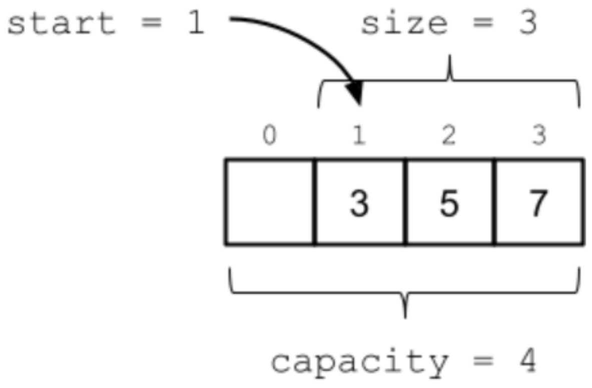
Implement Queue using Dynamic Array



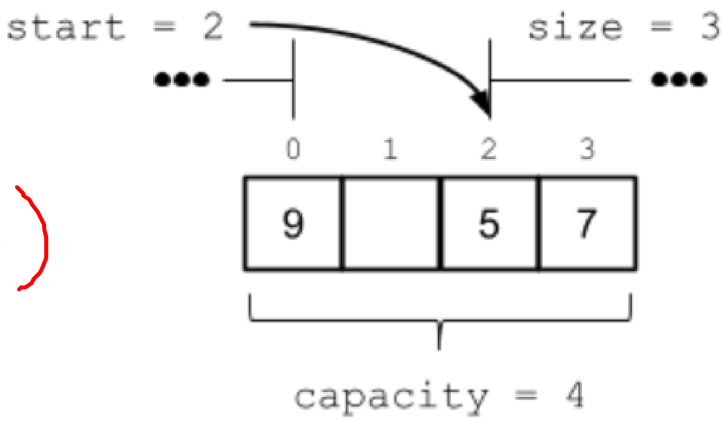
enqueue(7)



enqueue(9)



dequeue()



Implement Queue using Dynamic Array

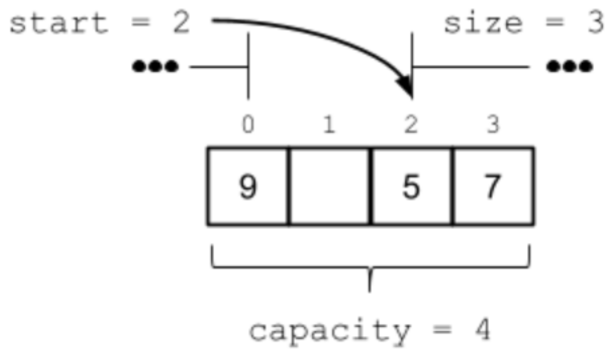
- An array that allows data to wrap around from the back to the front is known as a circular buffer

- Q: How do we know which index corresponds to the back of the queue?
  - By computing a mapping between the array's *logical indices* and its *physical indices*
- Logical indices – the indices relative to the start of the data
- Physical indices – the indices relative to the start of the physical array

Implement Queue using Dynamic Array

- Mapping formula: `physical = start + logical;`
- Since it is circular, add the following to check:

```
if (physical >= capacity) {
        physical -= capacity;
}
```



- OR: **`physical = (start + logical) % capacity;`**

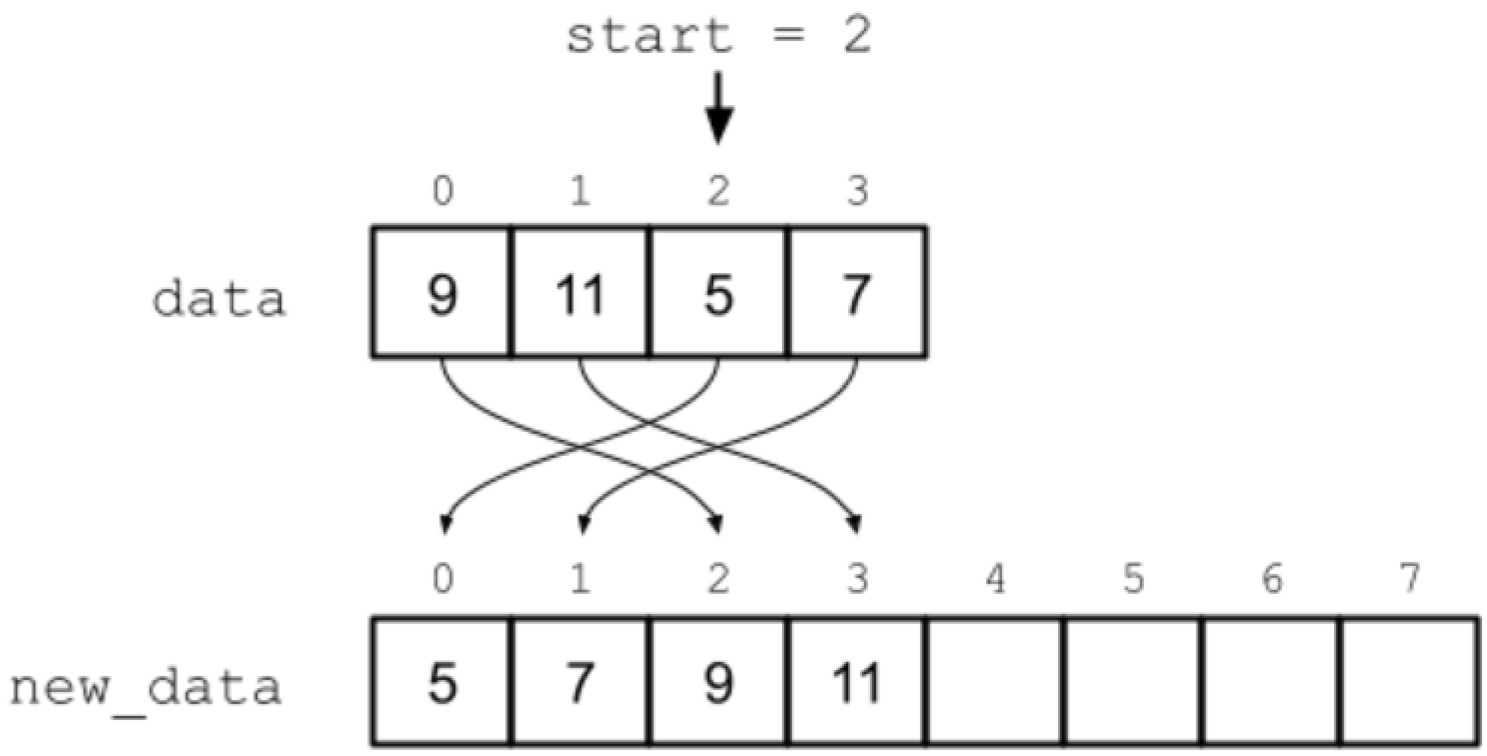
- Index at which the next element will be inserted:
  - Previously: array[size] – when the data starts at physical index 0
  - Now: array[physical] – where physical = (start + size) % capacity

Implement Queue using Dynamic Array

- Dynamic Array resizing for the queue implementation
- When do we need to resize?
  - size >= capacity
- When resize, reindex!
  - Logical index 0 ←→ Physical index 0
- How?
  - Loop through the logical indices from 0 to size – 1
  - Copy elements at each logical index in the old array to the equivalent physical index in the new array

Implement Queue using Dynamic Array

- Visually, look like this:



Implement Queue using Dynamic Array

- Complexity:
  - Dequeue – O(1) for all best-case, worst-case, and average case

  - Enqueue
    - O(1) for best-case and average case
    - O(n) for worst-case, when resize is needed